

电路原理I

复习课

电路

集总参数

线性电路

稳态电路

非线性电路

暂态电路

分布参数电路

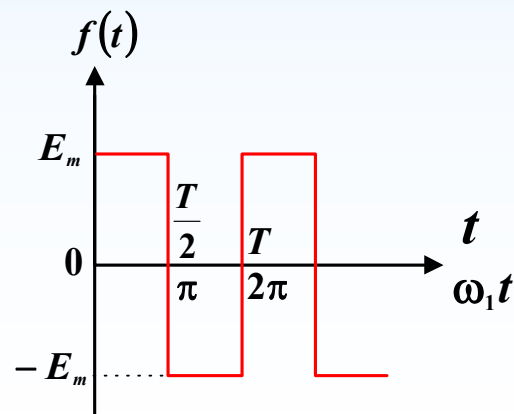
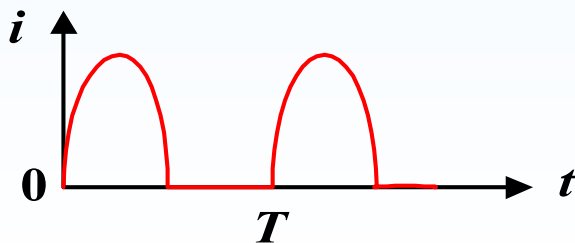
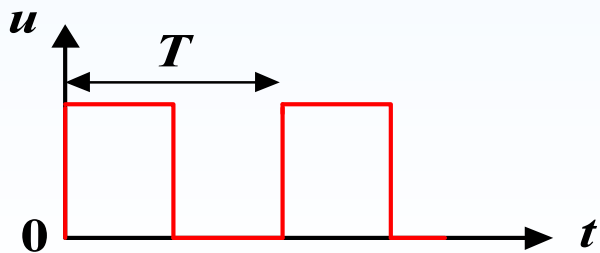
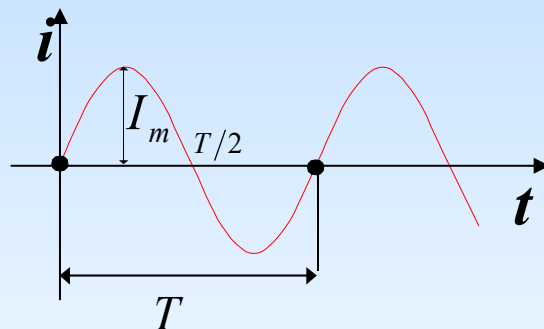
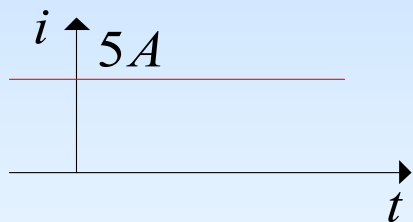
直流电路

正弦稳态交流电路

单相

三相

其他

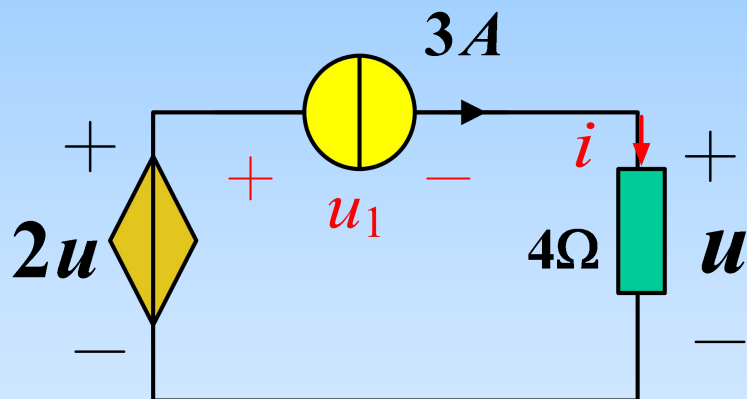


稳态电路

第一章 电路元件和电路定律 基础

注意：





电路如图，求各元件的功率。

解：

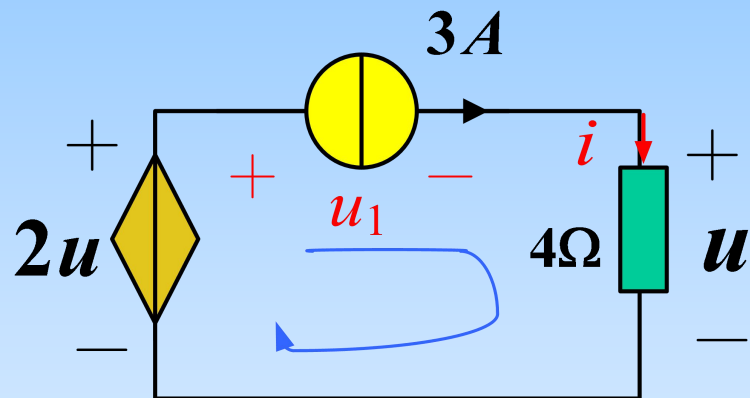
1、参考方向：每个元件电压电流是关联还是非关联？

2、元件VCR：电压电流关系。

电阻： $u = 4 \times 3 = 12V$

电流源：电流3A、电压 u_1 由外电路决定。

电压控制的电压源：电压 $2u$ ，由控制量 u 决定，电流 i 由外电路决定，电流3A。



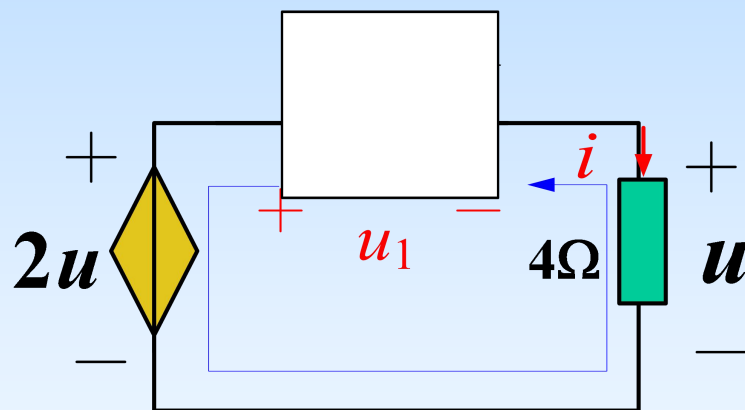
3、KCL、KVL。

KVL $\sum u(t) = 0$

$$u_1 + u - 2u = 0$$

$$u = 4 \times 3 = 12V$$

$$u_1 = u = 12V$$

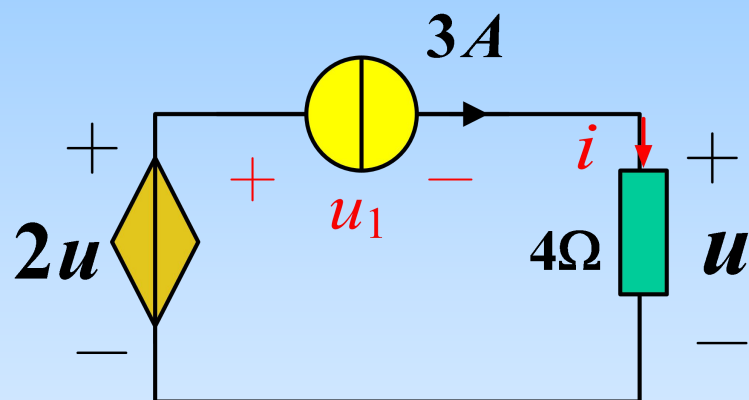


$$u_1 = 2u - u = 12V$$

推论：

电路中任意两点间的电压等于两点间任一条路径经过的各元件电压的代数和。元件电压方向与路径绕行方向一致时取正号，相反取负号。

4、功率计算和判断。



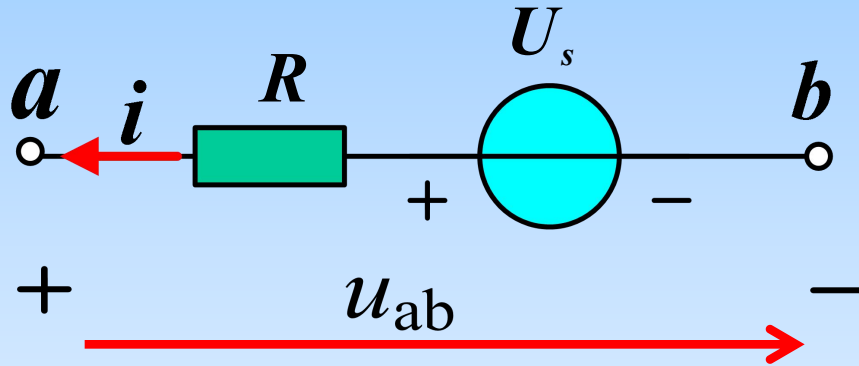
$$P_R = 12 \times 3 = 36W \quad \text{吸收}$$

$$P_{i_s} = u_1 \times 3 = 36W \quad \text{吸收}$$

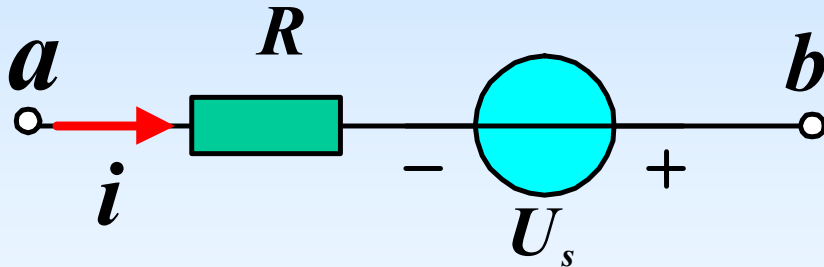
$$P_{\text{受控源}} = 2u \times 3 = 72W \quad \text{发出}$$

$$P_{\text{吸收}} = P_{\text{发出}} \quad \text{功率守恒}$$

求下列各电压 u_{ab} 与电流 i 的关系式。



解: $u_{ab} = -Ri + U_s$



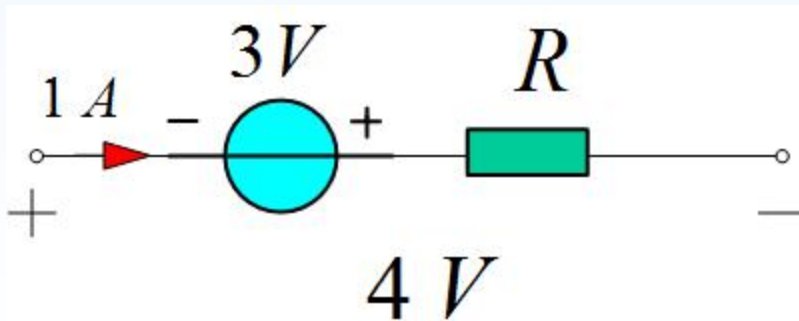
$u_{ab} = Ri - U_s$

求电路中的电阻 $R = (\quad)$ 。

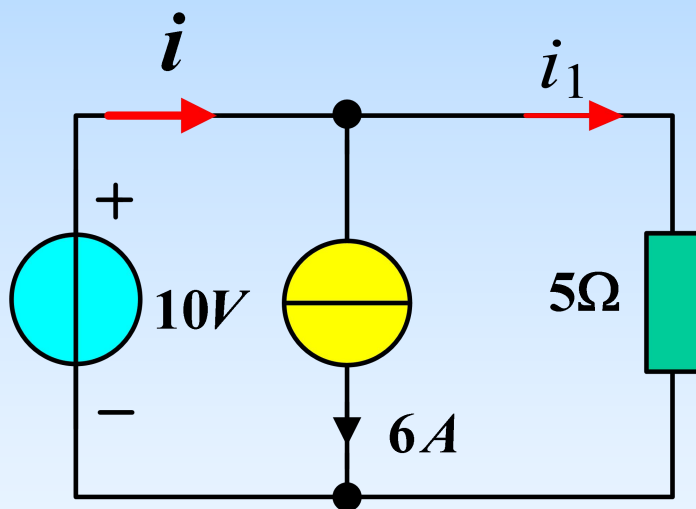
解:

$4 = -3 + R \times 1$

$R = 7\Omega$



求电流 i 。



解：

$$i_1 = \frac{10}{5} = 2A$$

KCL $i_1 + 6 - i = 0$

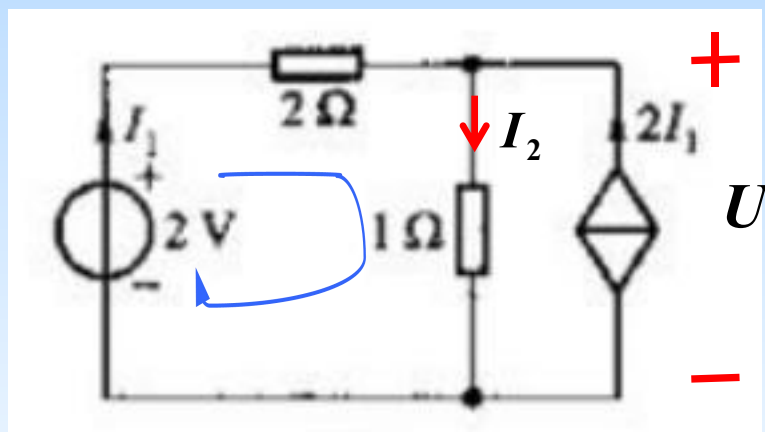
$$i = 8A$$

$$\sum i_{\lambda} = \sum i_{\text{出}}$$

$$i = i_1 + 6$$



图中电压源功率 P_1 、受控源功率 P_2 、电阻功率 P_3 ，并判断每个元件是发出还是吸收功率。



解: $KCL \quad I_2 = I_1 + 2I_1$

$KVL \quad 2 \times I_1 + 1 \times I_2 - 2 = 0$

求得 $I_1 = 0.4A$

$I_2 = 1.2A$

电压源 $P_1 = 2 \times I_1 = 0.8W$ (发出)

受控电流源

$P_2 = U \times 2I_1 = 0.96W$ (发出)

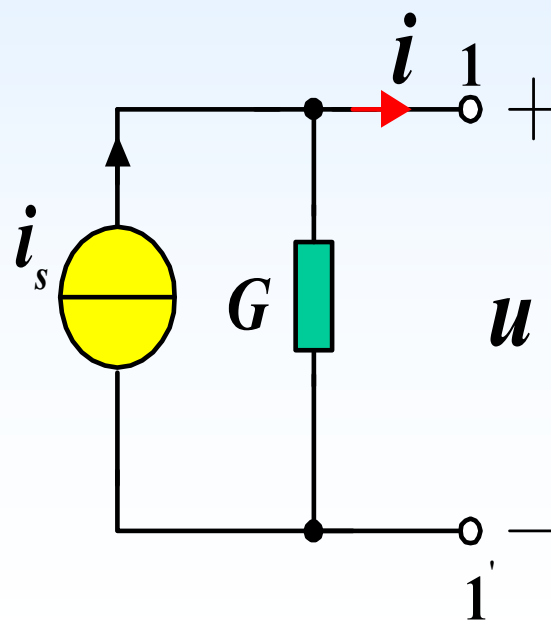
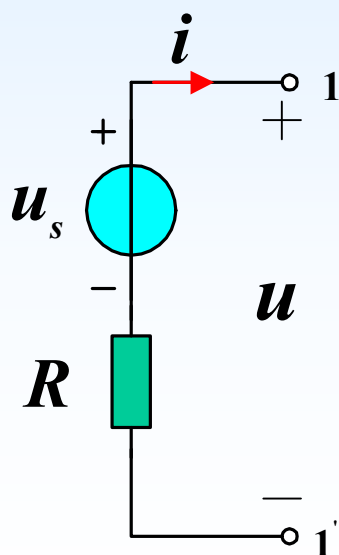
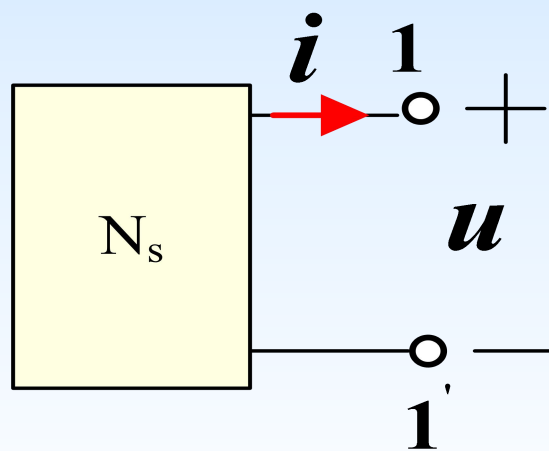
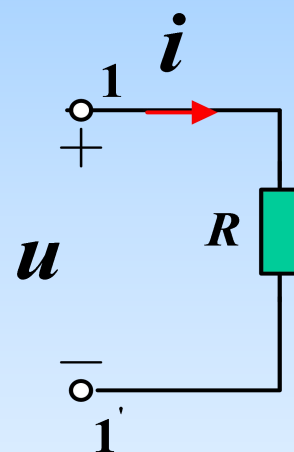
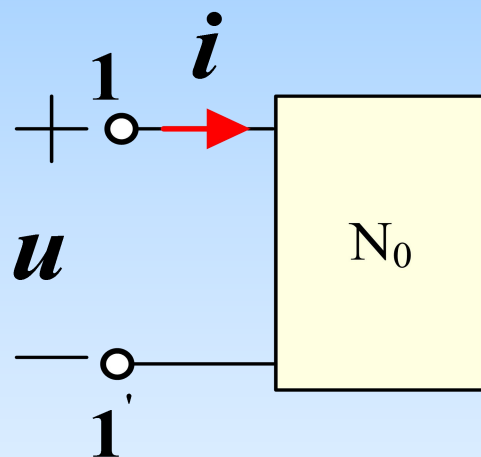
电阻

$P_3 = 0.32 + 1.44 = 1.76W$ (吸收)

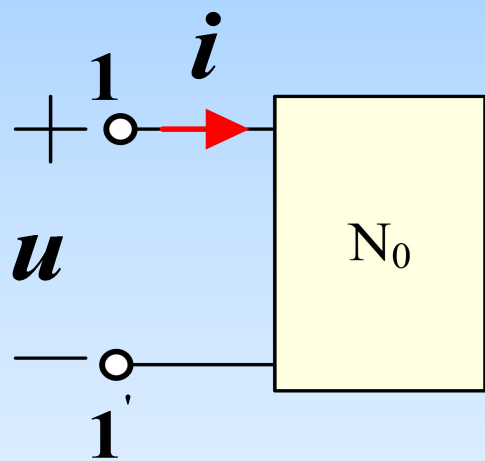
第二章 电阻电路的等效变换

等效化简

无源电路网络
有源电路网络



无源电路网络

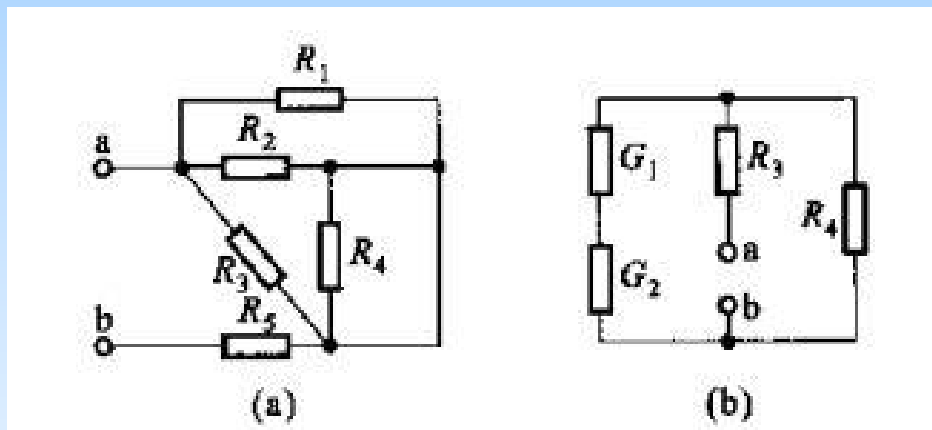


- 1、电阻串联、并联、混联
- 2、电阻的星形联接、三角形联接
- 3、含有受控源

电桥平衡、对称等

- 注意：
- 1、电阻串联分压公式
 - 2、电阻并联分流公式
 - 3、含有受控源等效电阻计算

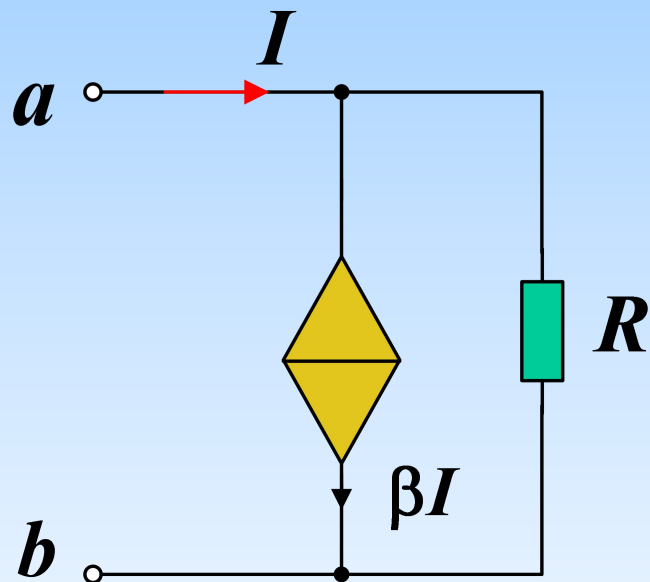
$R_1 = R_2 = 1\Omega$, $R_3 = R_4 = 2\Omega$, $R_5 = 4\Omega$, 求等效电阻 R_{ab} 。



解:
$$R_{ab} = R_5 + R_1 // R_2 // R_3 = 4 + \frac{1}{1 + 1 + \frac{1}{2}} = 4.4 \Omega$$

$$R_{ab} = R_3 + \left(\frac{1}{G_1} + \frac{1}{G_2} \right) // R_4$$

求 a,b 两端的输入电阻 R_{ab} ($\beta \neq 1$)



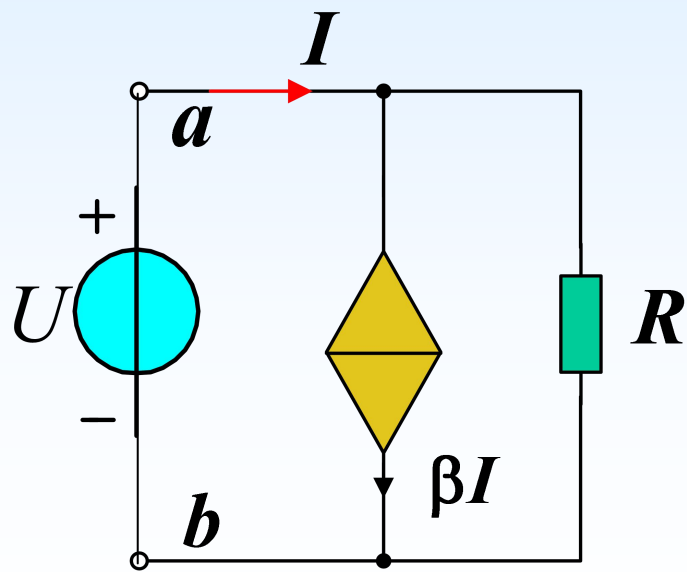
解：通常有两种求输入电阻的方法

① 加压求流法

② 加流求压法

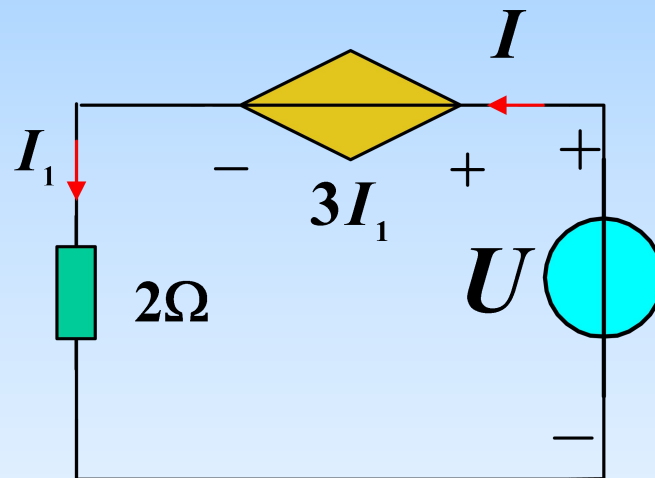
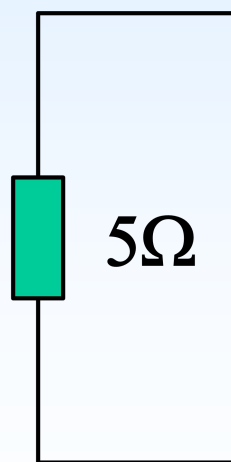
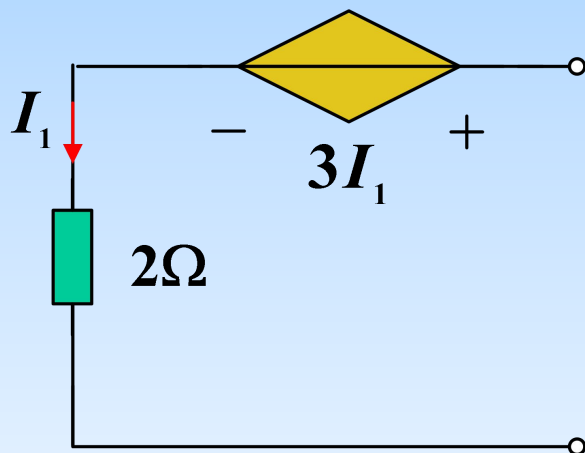
$$R_{ab} = \frac{U}{I}$$

$$U = R(I - \beta I)$$



$$R_{ab} = \frac{U}{I} = R(1 - \beta)$$

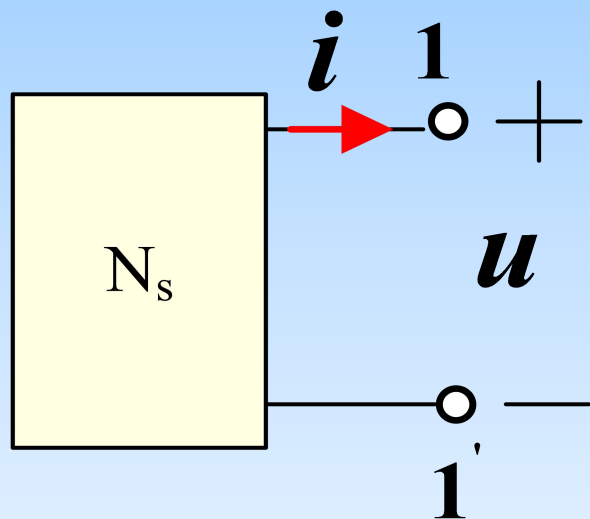
例4. 简化电路:



$$U = 3I_1 + 2I_1$$

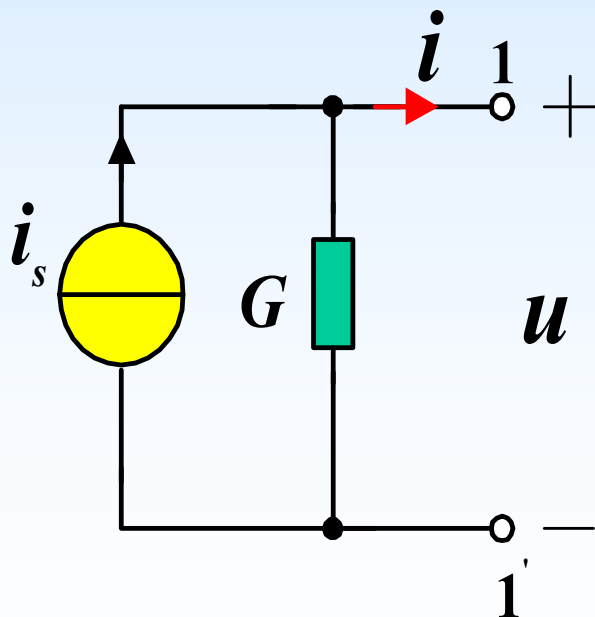
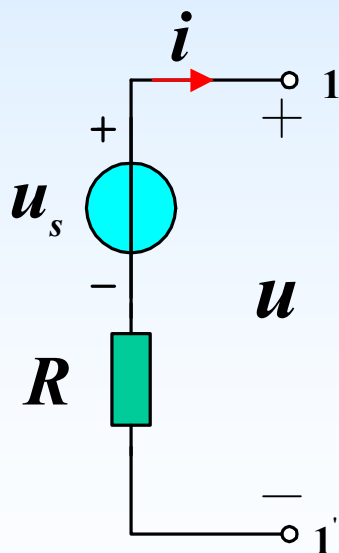
$$= 5I$$

$$R = \frac{U}{I} = 5\Omega$$

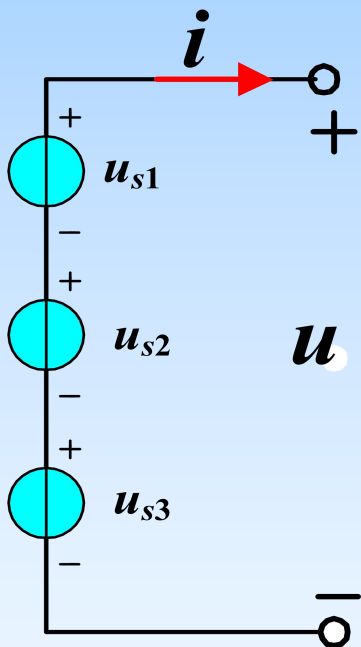


- 1、电压源串联
- 2、电流源并联
- 3、电压源电阻串联、电流源电阻并联

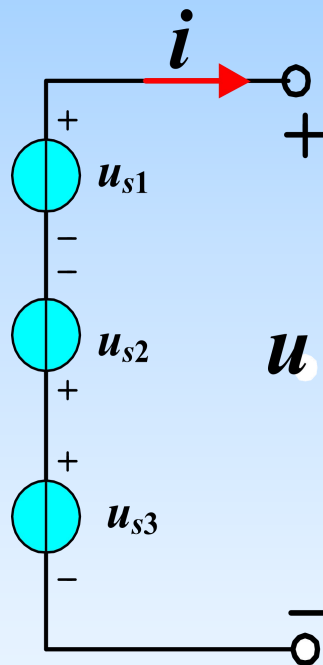
等效化简



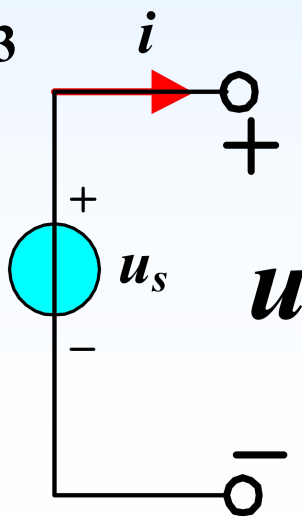
理想电压源的串联



$$u_s = u_{s1} + u_{s2} + u_{s3}$$

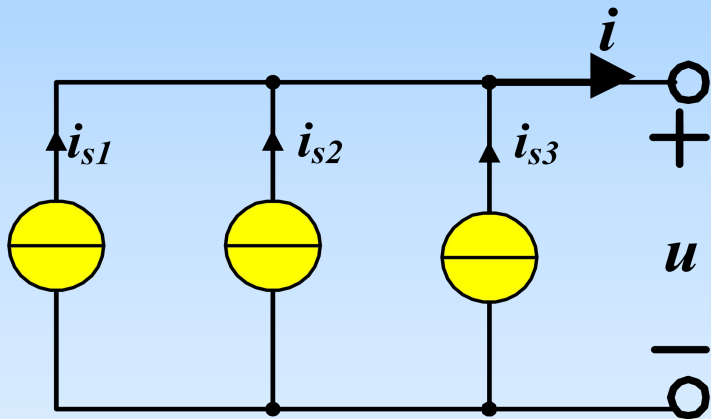


$$u_s = u_{s1} - u_{s2} + u_{s3}$$

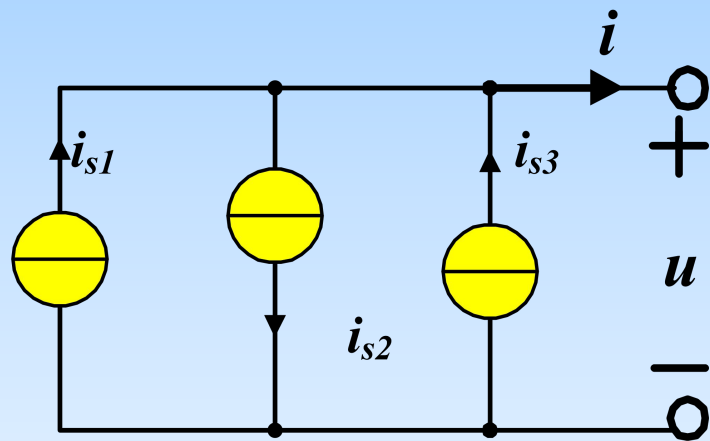


注意：电压源参考方向

理想电流源的并联

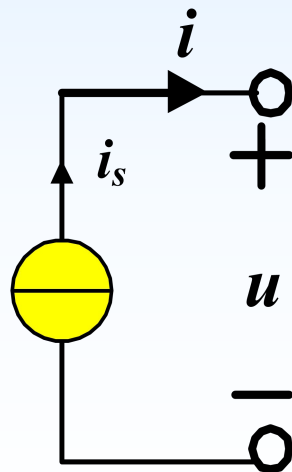


$$i_s = i_{s1} + i_{s2} + i_{s3}$$

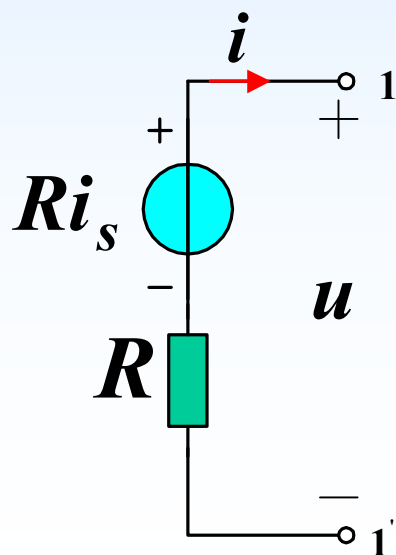
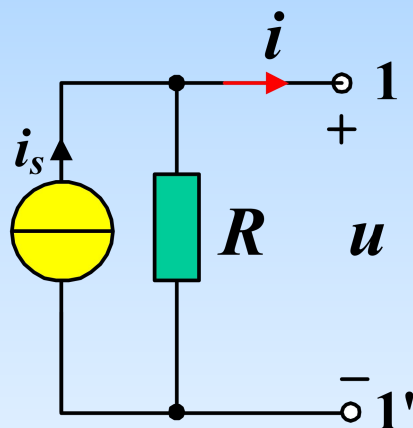
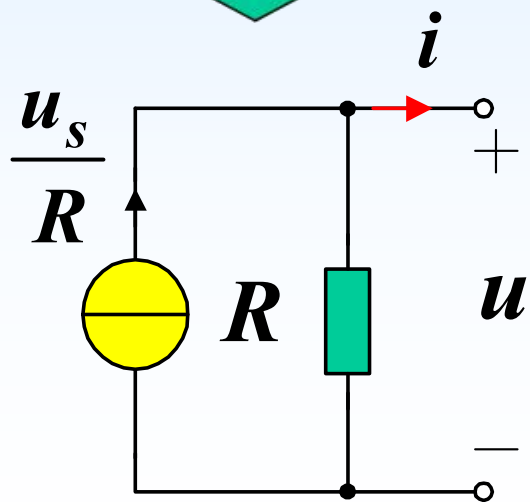
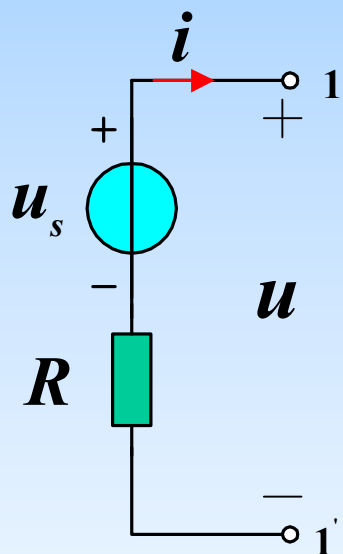


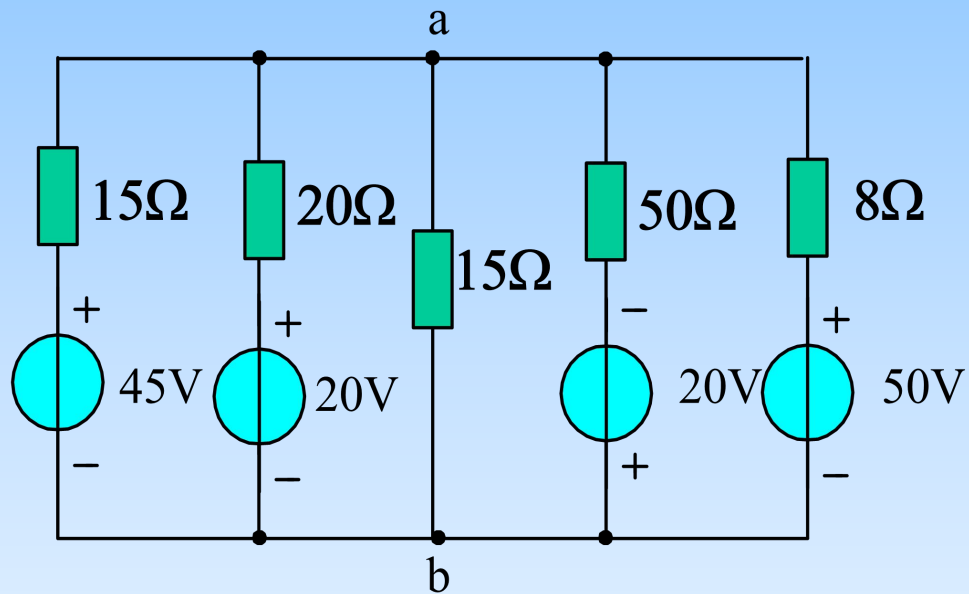
$$i_s = i_{s1} - i_{s2} + i_{s3}$$

注意： 电流源参考方向

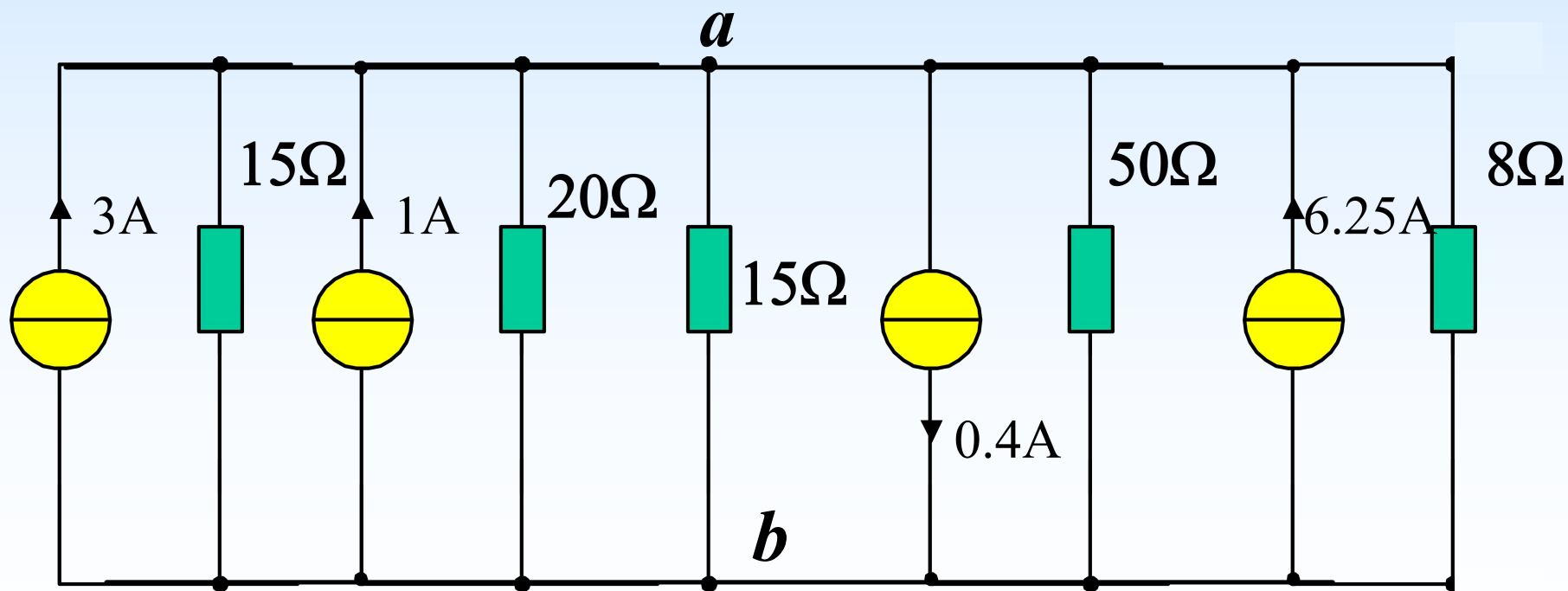


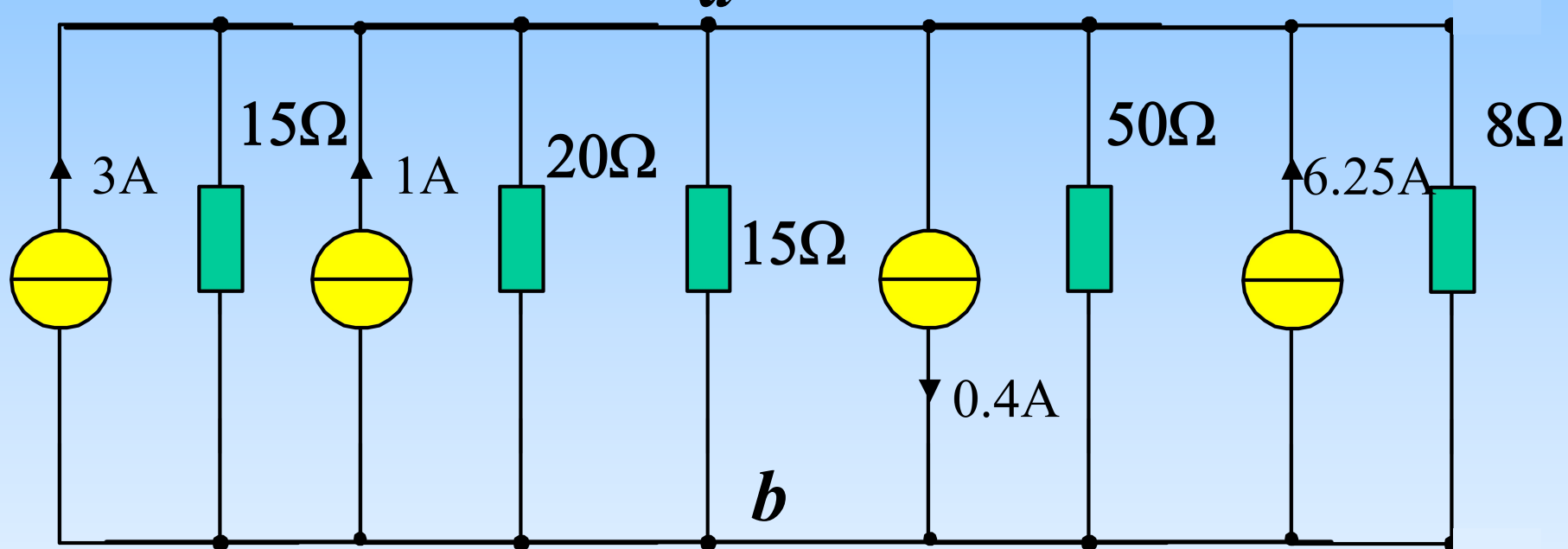
实际电源的两种模型及其等效变换



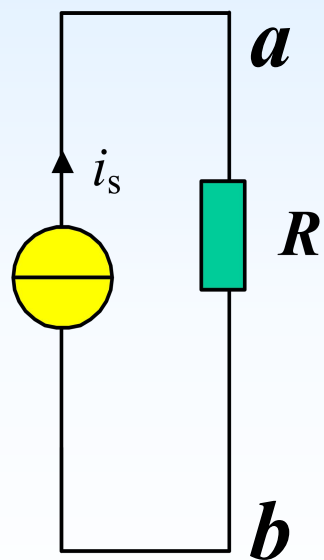


用电源等效变换的方法求
电压 u_{ab} 。





$$i_s = 3 + 1 - 0.4 + 6.25 = 9.85 A$$



$$R = \frac{1}{\frac{1}{15} + \frac{1}{20} + \frac{1}{15} + \frac{1}{50} + \frac{1}{8}} = 3.05 \Omega$$

$$u_{ab} = 3.05 \times 9.85 = 30 V$$

第三章 电阻电路的一般分析方法

电路
分析方法

{

- 支路法
- 回路电流法（网孔电流法）
- 结点电压法

n个结点
b条支路

方程数

	KCL方程	KVL方程	方程总数
支路法	$n-1$	$b-(n-1)$	b
回路法	0	$b-(n-1)$	$b-(n-1)$
结点法	$n-1$	0	$n-1$

回路（网孔）
电流法

{ 电阻、电压源
含有无伴电流源
受控源

假设无伴电流源为某一个回路所有

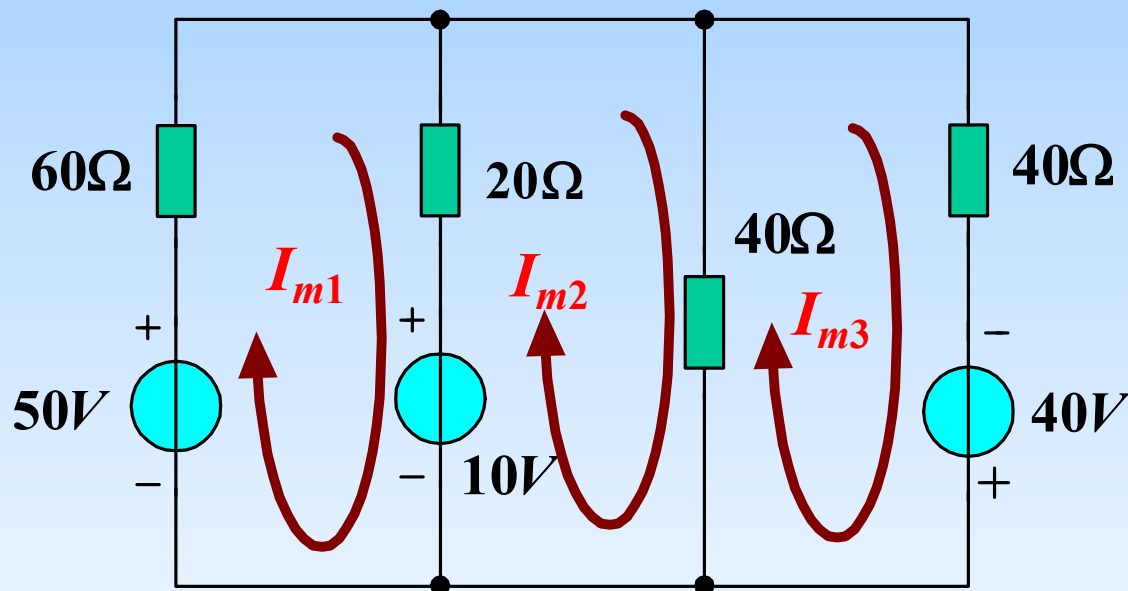
（1）先把它当成独立源看待。

（2）再列写补充方程，写出控制量和回路（网孔）电流关系。

对独立回路，列KVL方程；（自阻(正)、互阻（正、负、零）、电压源方向与回路（网孔）电流方向一致时，前面取负号；反之取正号。

自电阻*本回路电流+ $\sum$ 互电阻*相邻回路电流 = $\sum$ 电压源的代数和

用网孔电流法求各支路电流。



解:

(1) 设网孔电流(顺时针) I_{m1} I_{m2} I_{m3}

$$(60 + 20) I_{m1} - 20 I_{m2} = 50 - 10$$

(2) 列网孔方程

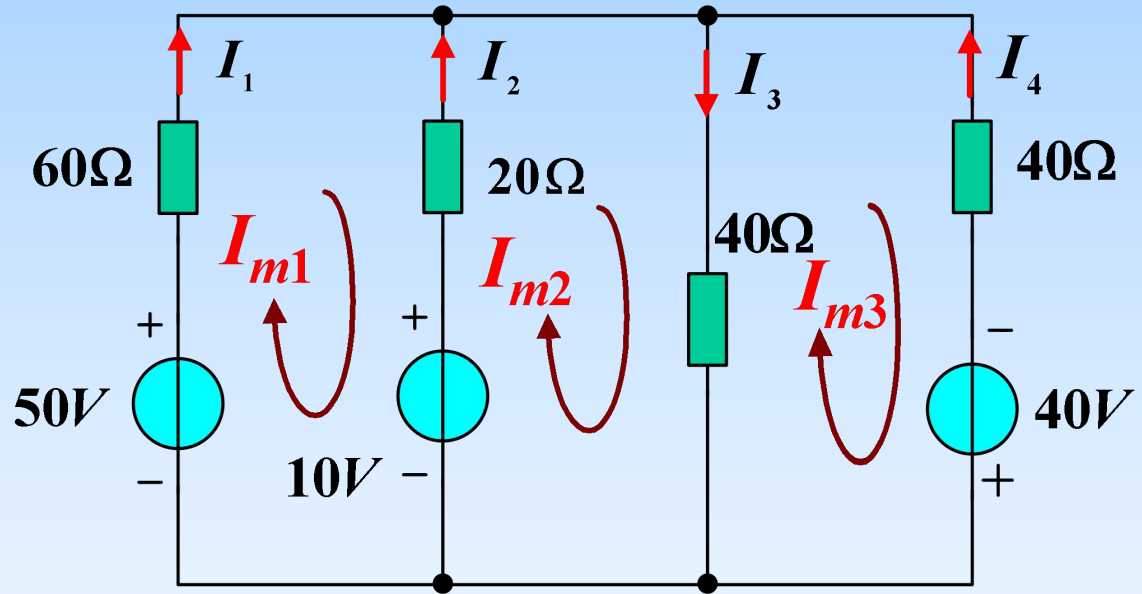
$$- 20 I_{m1} + (20 + 40) I_{m2} - 40 I_{m3} = 10$$

$$- 40 I_{m2} + (40 + 40) I_{m3} = 40$$

(3) 求解网孔电流方程，得 I_{m1} , I_{m2} , I_{m3}

得:

$$I_{m1} = 0.786\text{A}$$
$$I_{m2} = 1.143\text{A}$$
$$I_{m3} = 1.071\text{A}$$

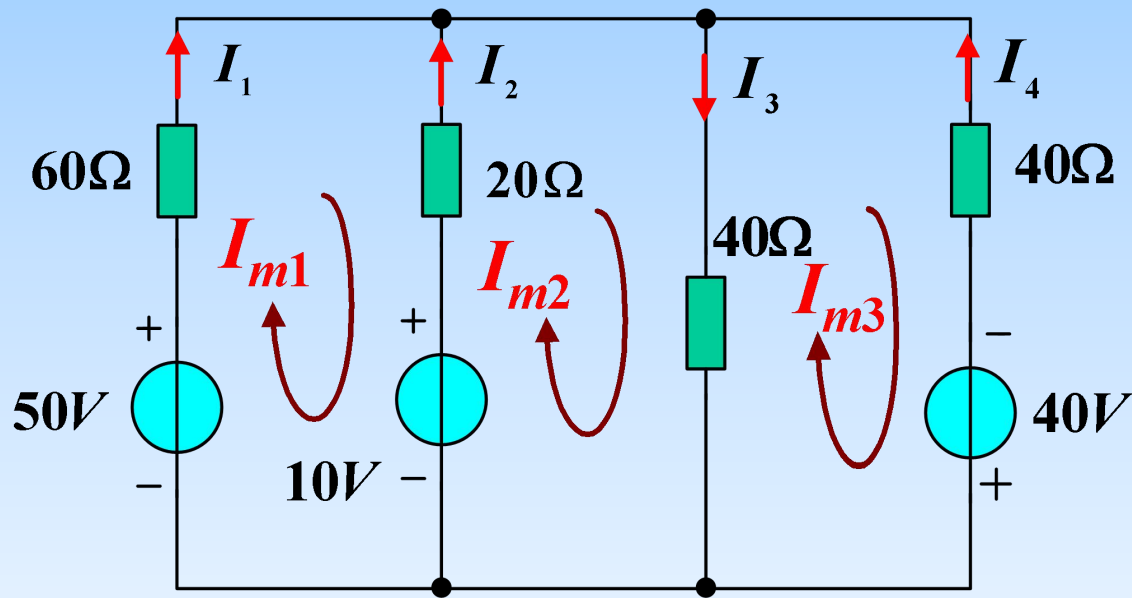


(4) 求各支路电流: $I_1 = I_{m1} = 0.786\text{A}$

$$I_2 = -I_{m1} + I_{m2} = 0.357\text{A}$$

$$I_3 = I_{m2} - I_{m3} = 0.072\text{A}$$

$$I_4 = -I_{m3} = -1.071\text{A}$$



$$I_{m1} = 0.786\text{A}$$

$$I_{m2} = 1.143\text{A}$$

$$I_{m3} = 1.071\text{A}$$

$$I_1 = I_{m1} = 0.786\text{A}$$

$$I_2 = -I_{m1} + I_{m2} = 0.357\text{A}$$

$$I_3 = I_{m2} - I_{m3} = 0.072\text{A}$$

$$I_4 = -I_{m3} = -1.071\text{A}$$

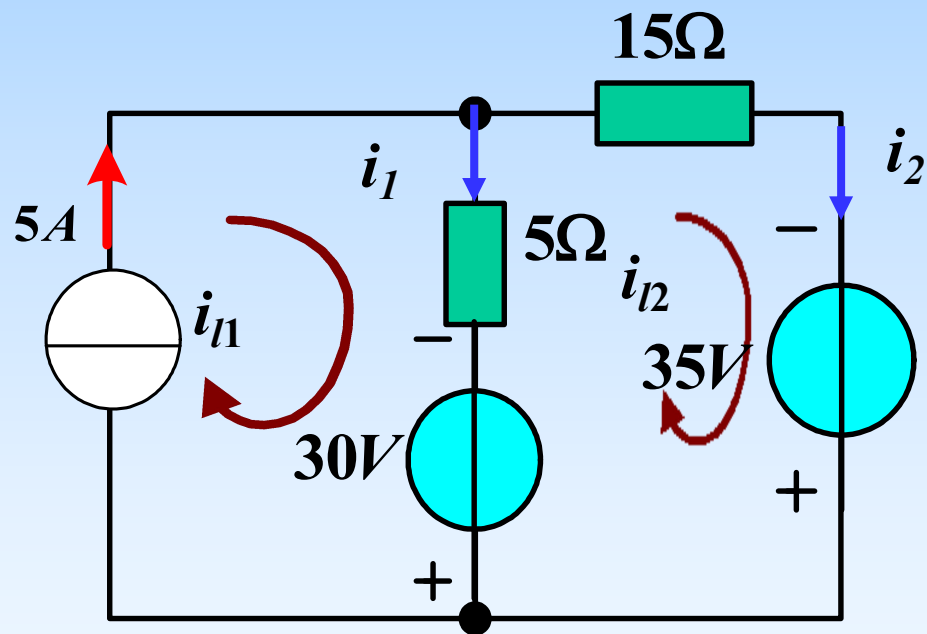
求各支路电流 i_1 、 i_2 。

$$i_{l1}=5$$

$$-5i_{l1} + 20i_{l2} = 35 - 30$$

$$i_2 = i_{l2} = 1.5\text{A}$$

$$i_1 = i_{l1} - i_{l2} = 3.5\text{A}$$



结点电压法 { 电阻、电流源
 { 含有无伴电压源
 { 受控源

假设无伴电压源为某一结点电压。

(1) 先把它当成独立源看待。

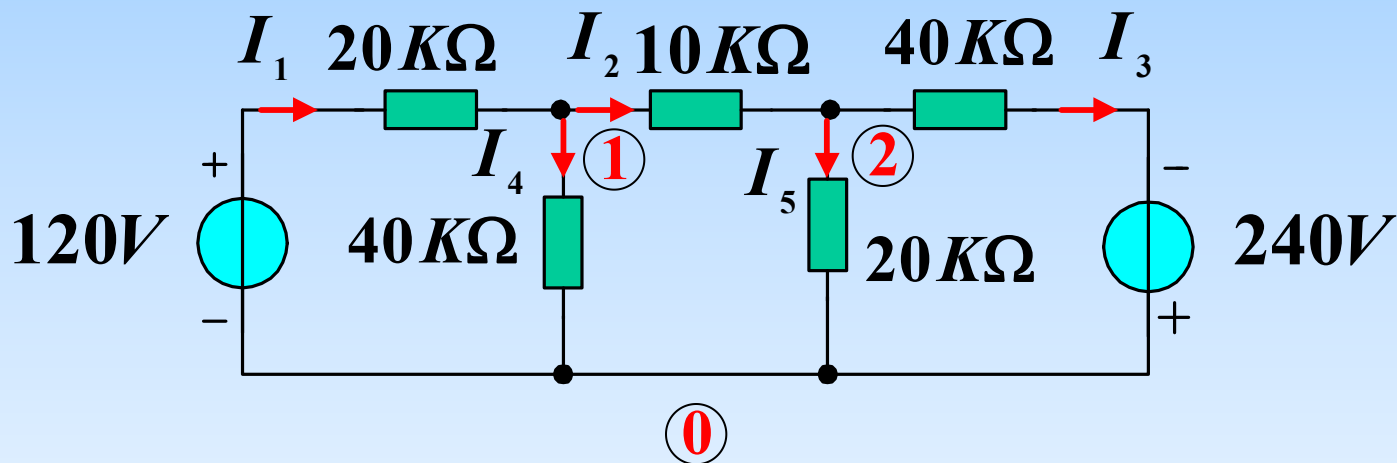
(2) 再列写补充方程，把控制量用结点电压表示。

对 $n-1$ 个独立结点，列写KCL方程；**自导**（正）、**互导**（负）、**电流源**（流入结点取“正”，流出结点取“负”）。

自电导*本结点电压 + $\sum$ 互电导*相邻结点电压 =

$\sum$ 本结点电流源的代数和

用结点法求各支路电流。

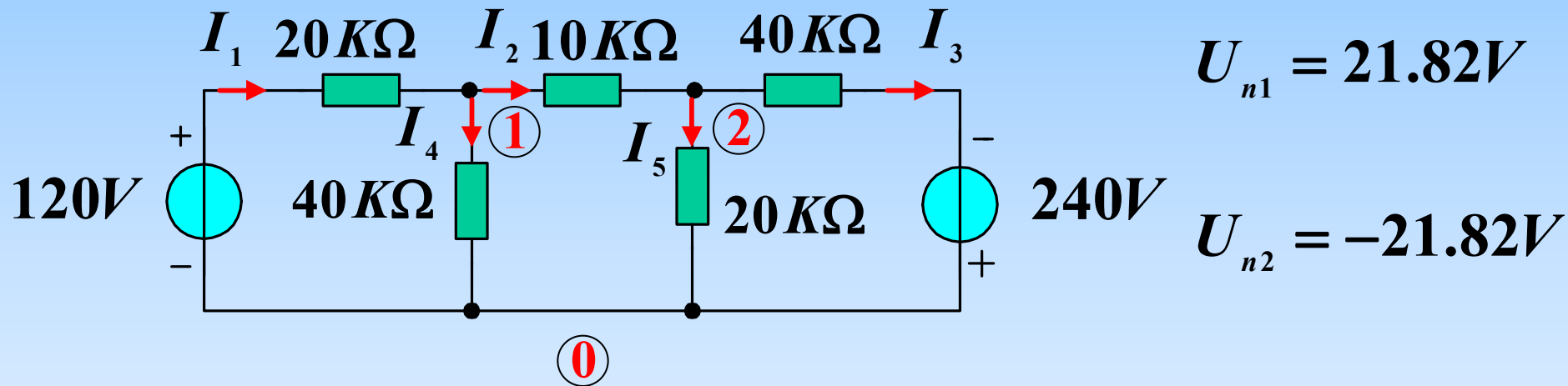


解:

$$\begin{cases} \left(\frac{1}{20 \times 10^3} + \frac{1}{40 \times 10^3} + \frac{1}{10 \times 10^3} \right) U_{n1} - \frac{1}{10 \times 10^3} U_{n2} = \frac{120}{20 \times 10^3} \\ -\frac{1}{10 \times 10^3} U_{n1} + \left(\frac{1}{10 \times 10^3} + \frac{1}{20 \times 10^3} + \frac{1}{40 \times 10^3} \right) U_{n2} = -\frac{240}{40 \times 10^3} \end{cases}$$

$$U_{n1} = 21.82V$$

$$U_{n2} = -21.82V$$



$$I_1 = \frac{120 - U_{n1}}{20 \times 10^3} = 4.91mA$$

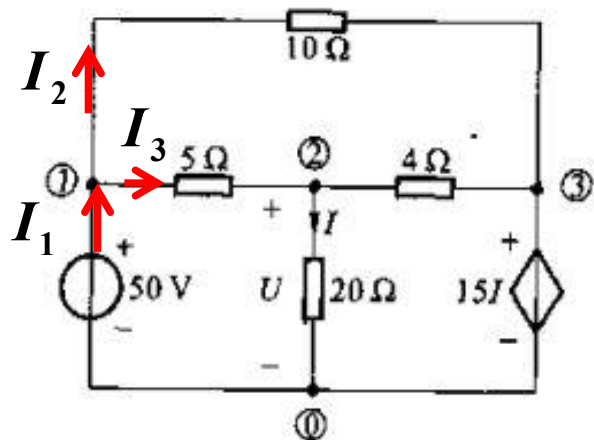
$$I_4 = \frac{U_{n1}}{40 \times 10^3} = 0.54mA$$

$$I_2 = \frac{U_{n1} - U_{n2}}{10 \times 10^3} = 4.36mA$$

$$I_5 = \frac{U_{n2}}{20 \times 10^3} = -1.09mA$$

$$I_3 = \frac{U_{n2} + 240}{40 \times 10^3} = 5.45mA$$

3-21 用结点电压法求结点电压 u_{n1} 、 u_{n2} 、 u_{n3} 和电压 U 。



$$u_{n1} = 50V$$

$$-\frac{1}{5}u_{n1} + \left(\frac{1}{5} + \frac{1}{20} + \frac{1}{40}\right)u_{n2} - \frac{1}{4}u_{n3} = 0$$

$$u_{n3} = 15I$$

$$I = \frac{u_{n2}}{20}$$

$$U = u_{n2} = 32V$$

$$I = \frac{u_{n2}}{20} = 1.6A$$

$$u_{n3} = 24V$$

$$I_1 = I_2 + I_3$$

$$= \frac{u_{n1} - u_{n3}}{10} + \frac{u_{n1} - u_{n2}}{5}$$

$$= 6.2A$$

第四章 电路定理

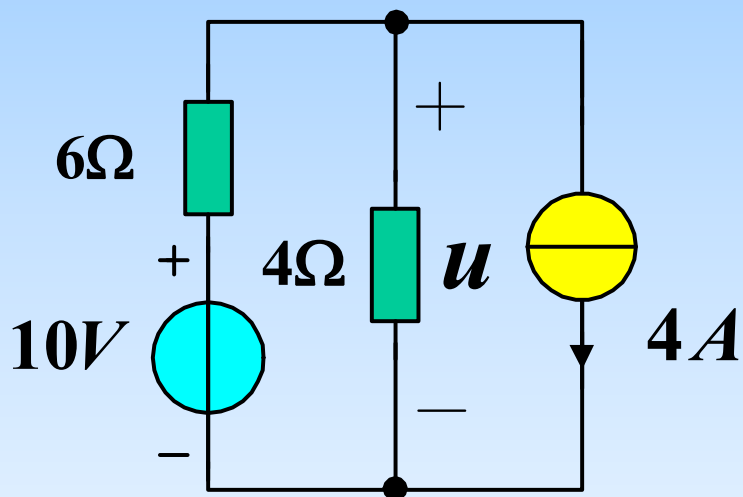
简化计算



叠加定理

在线性电路中，任一电流(或电压)都是电路中各个独立电源单独作用时，在该处产生的电流(或电压)的叠加（代数和）。

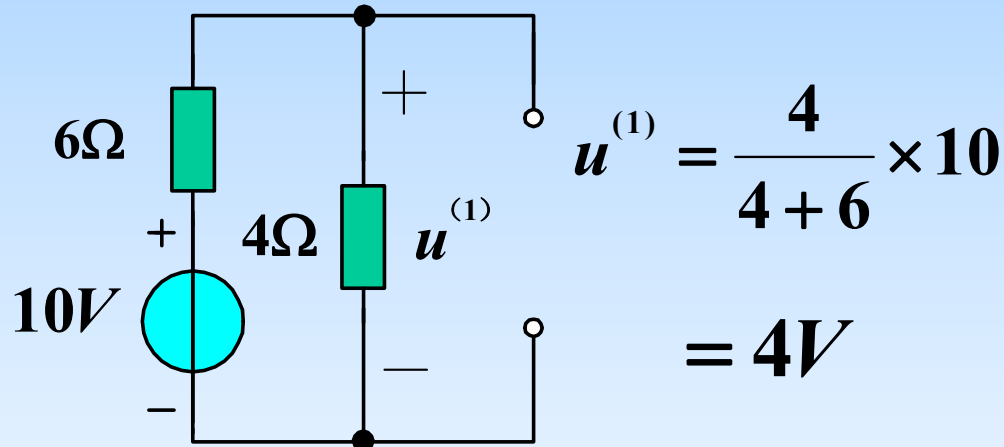
求图中电压 u 。



解:

(1) 10V电压源单独作用，

4A电流源开路

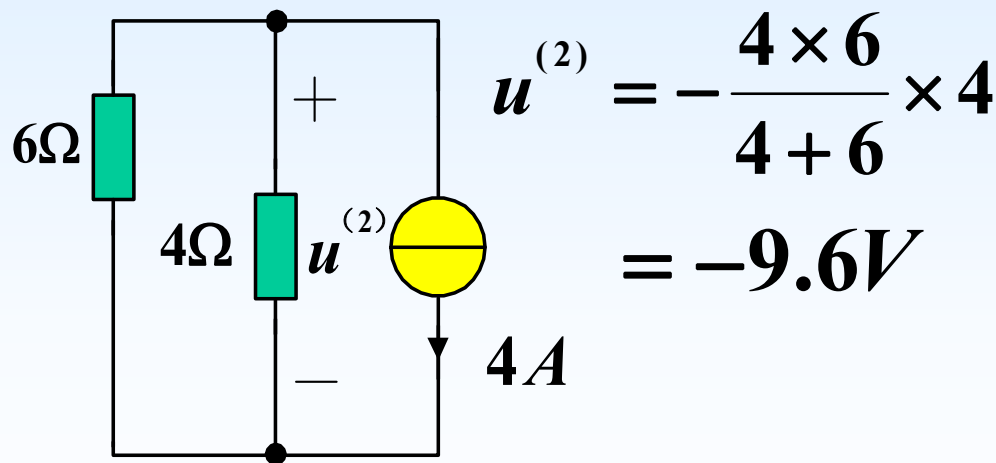


(2) 4A电流源单独作用，

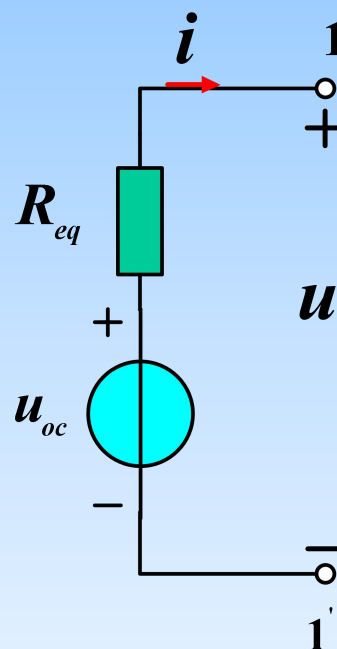
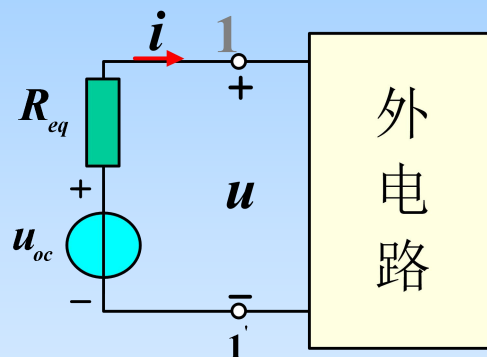
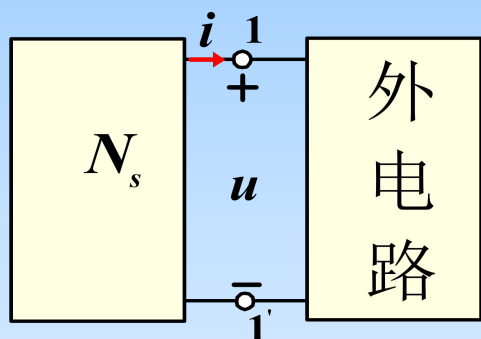
10V电压源短路

共同作用: $u = u^{(1)} + u^{(2)}$

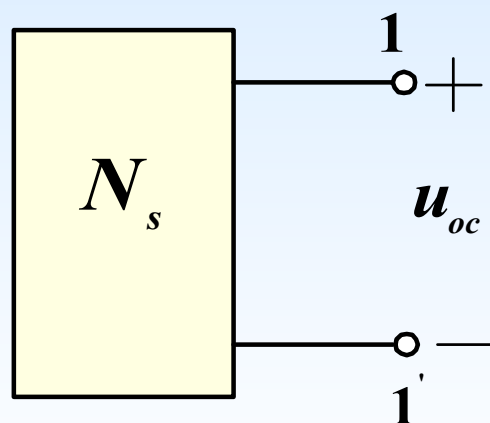
$$= 4 + (-9.6)$$
$$= -5.6V$$



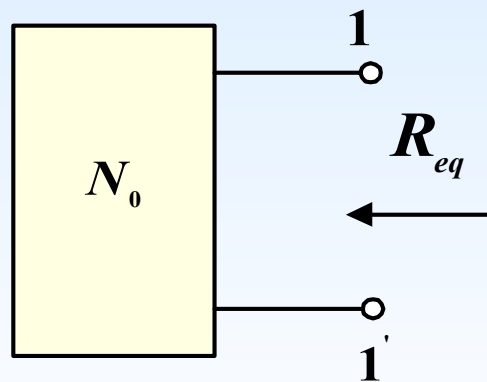
戴维宁定理和诺顿定理



戴维宁等效电路。

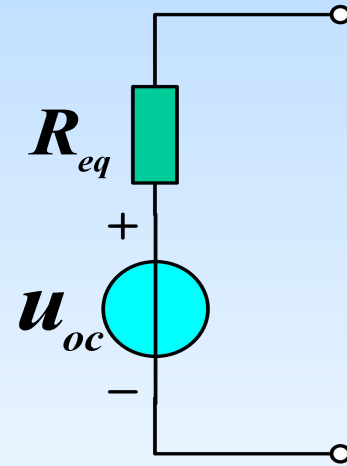
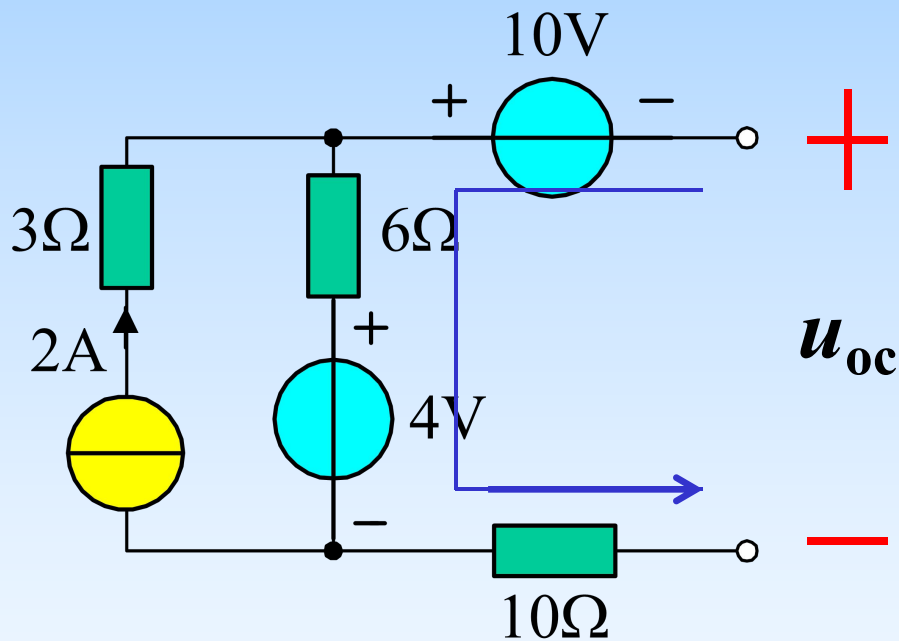


u_{oc} 开路电压。



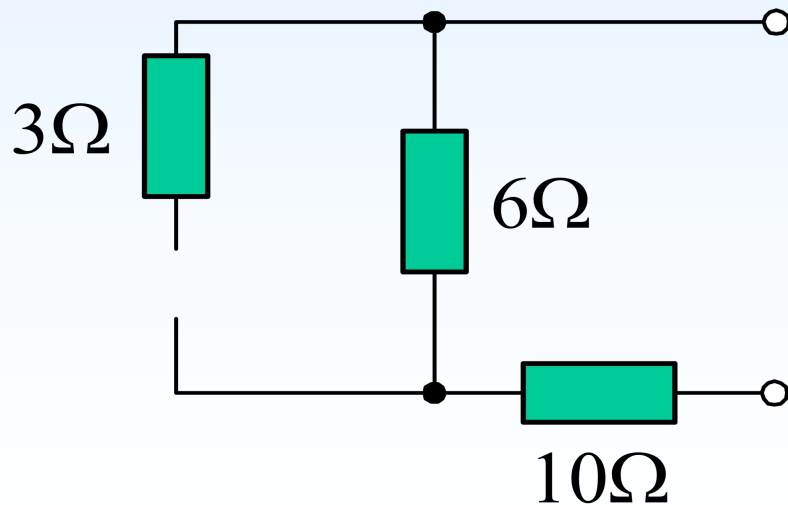
R_{eq} 称为戴维宁等效电阻。

求戴维宁等效电路。

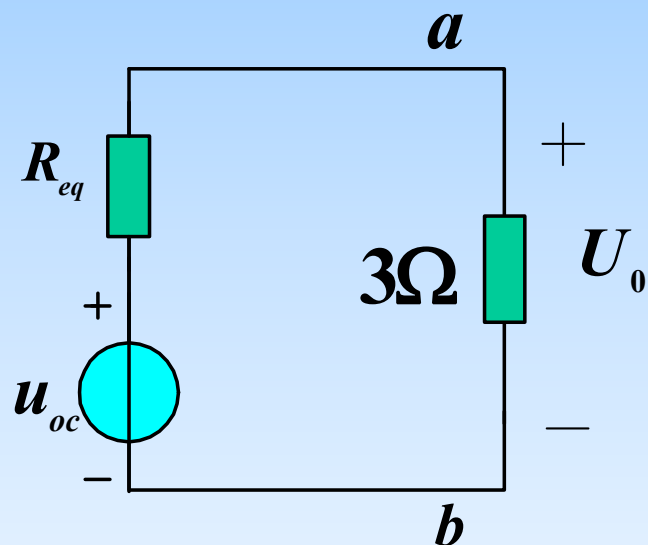
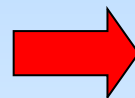
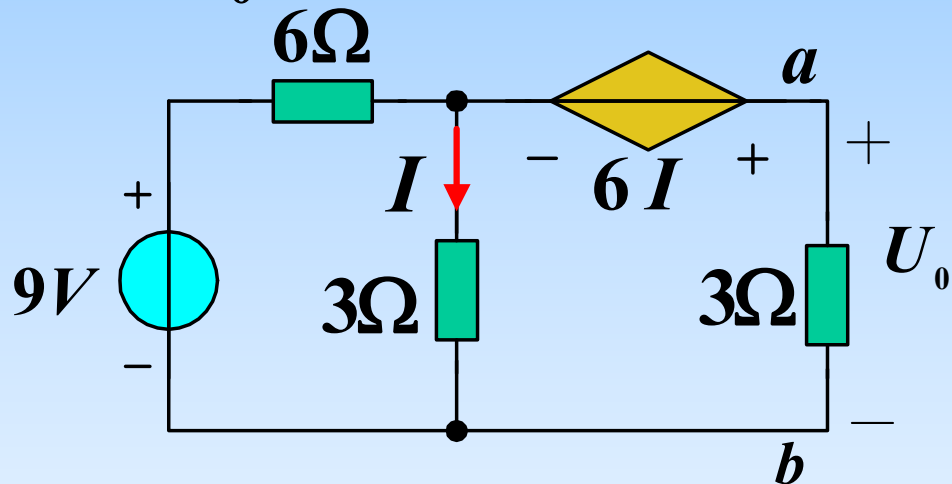


$$U_{oc} = -10 + 6 \times 2 + 4 = 6V$$

$$R_{eq} = 6 + 10 = 16\Omega$$

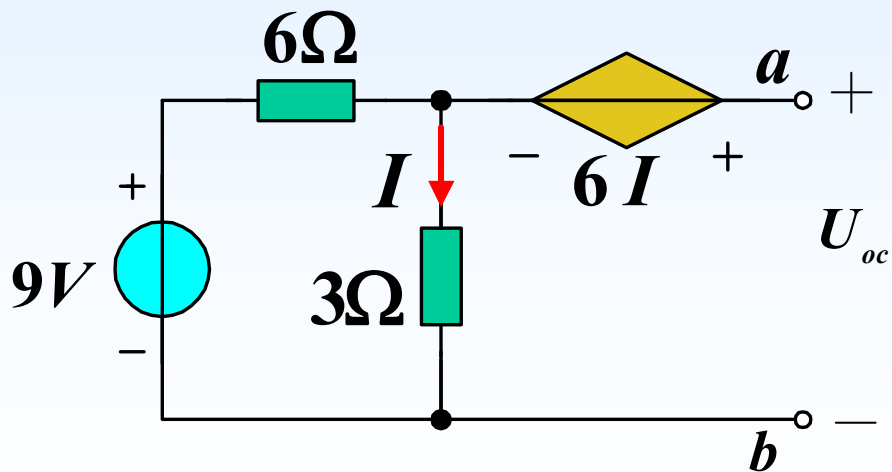


求 U_0 。



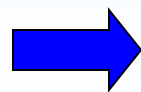
解:

(1) 求开路电压 U_{oc}



$$U_{oc} = 6I + 3I$$

$$I = \frac{9}{6 + 3} = 1A$$

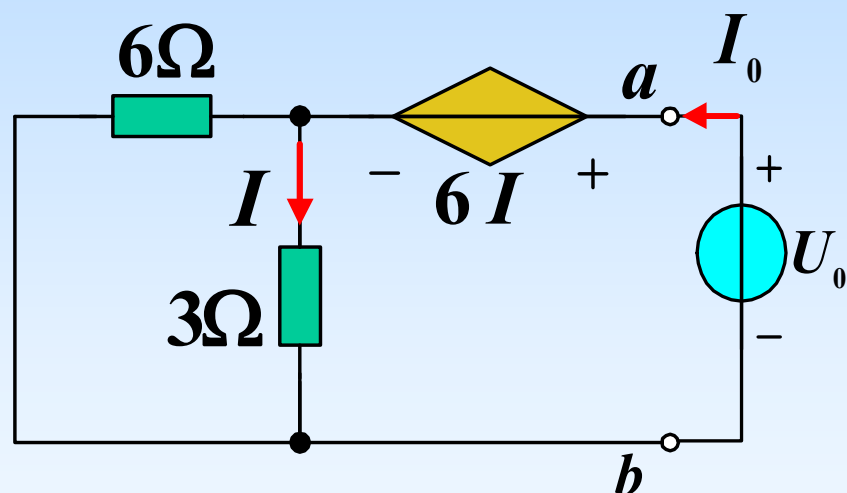
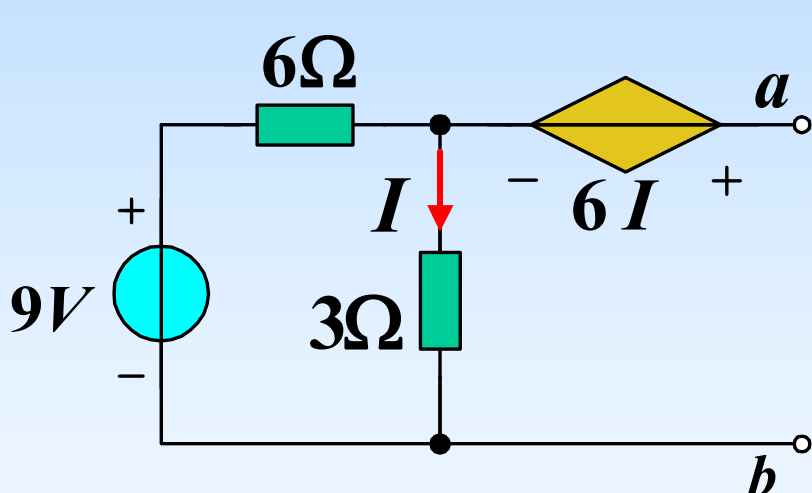


$$U_{oc} = 9V$$

(2) 求等效电阻 R_{eq}

加压求流法

将内部独立电源全部置零，外加独立电压源 U_0 ，求 $R_{eq} = \frac{U_0}{I_0}$



$$U_0 = 6I + 3I$$

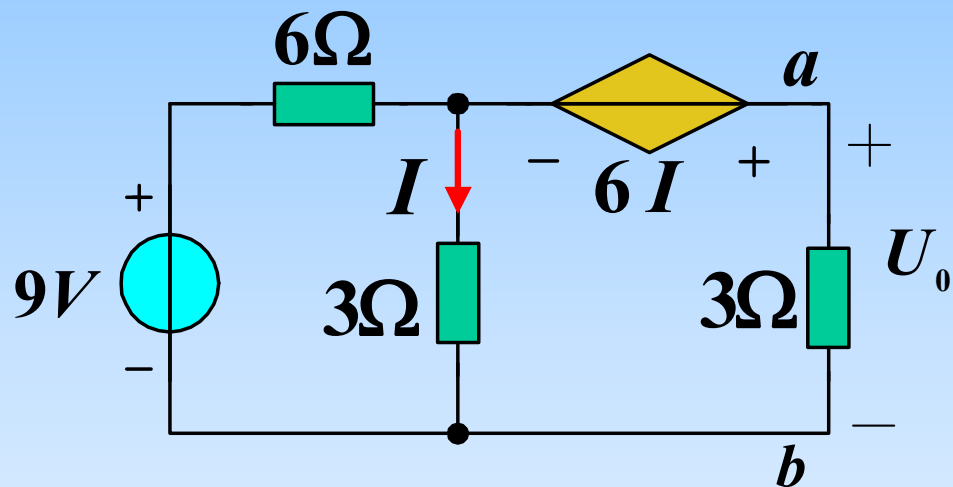
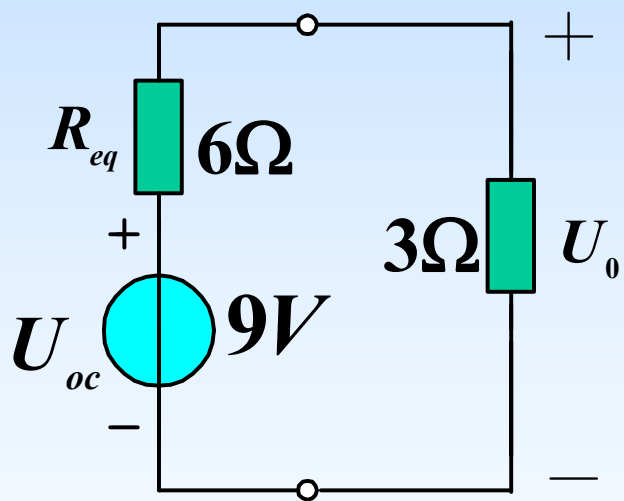
$$I = \frac{6}{6+3} I_0$$



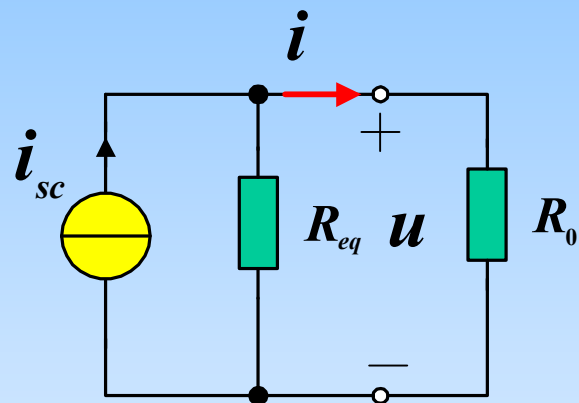
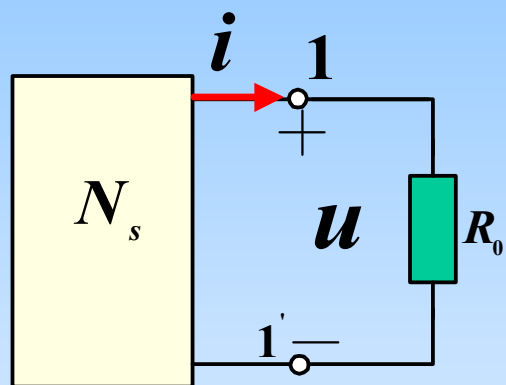
$$U_0 = 9 \times \frac{6}{9} I_0 = 6I_0$$

$$R_{eq} = \frac{U_0}{I_0} = 6\Omega$$

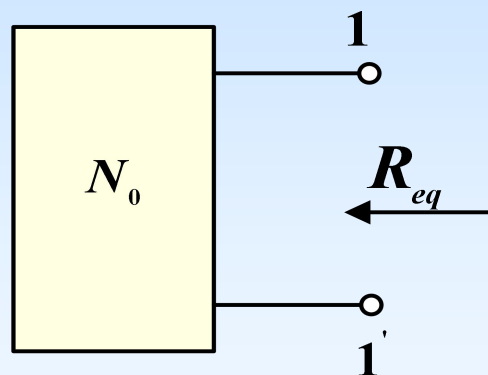
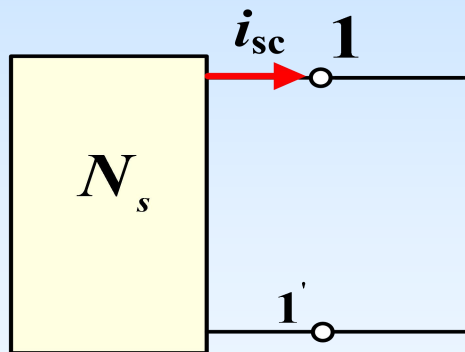
(3) 等效电路



$$U_0 = \frac{3}{6+3} \times 9 = 3V$$

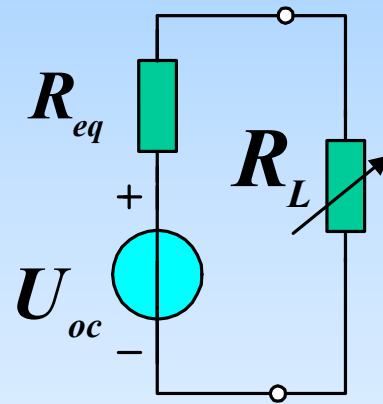
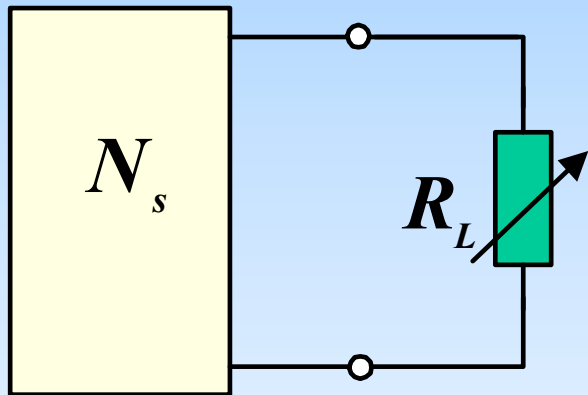


诺顿等效电路



诺顿等效电路可由戴维宁等效电路经电源等效变换得到。但须指出，诺顿等效电路可独立进行证明。证明过程从略。

最大功率传输定理:

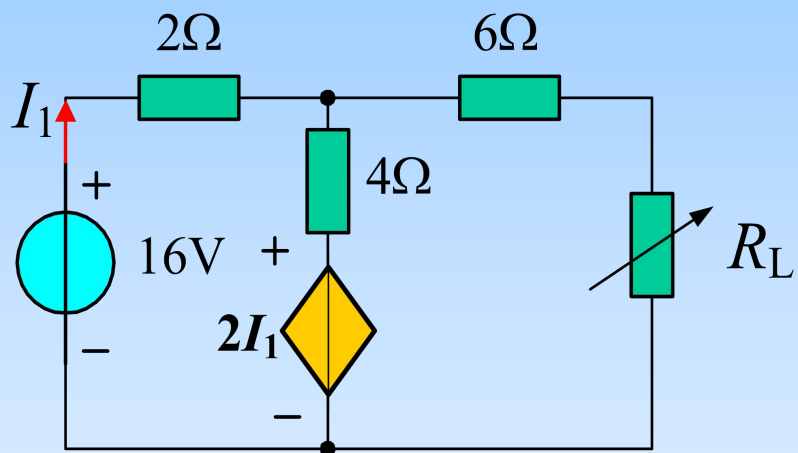


当 $R_L = R_{eq}$

最大功率 P_{max} 条件:

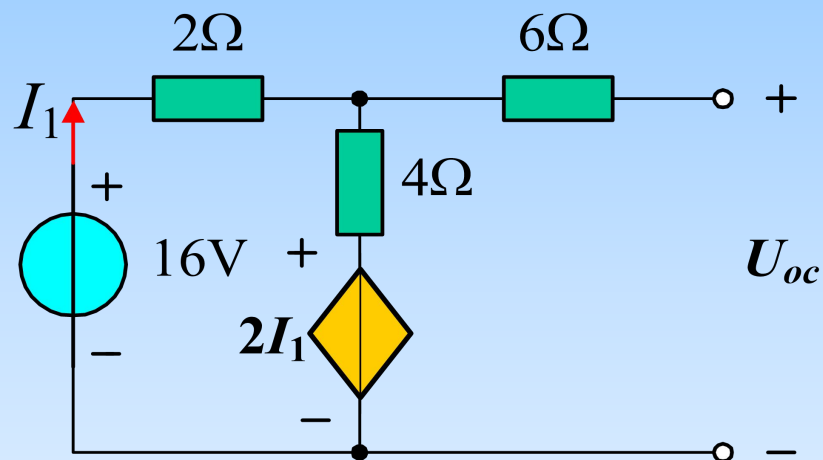
R_L 获得最大功率

$$P_{\max} = \frac{U_{oc}^2}{4R_{eq}}$$



R_L 为何值时，其上获最大功率？

1、求开路电压 U_{oc}

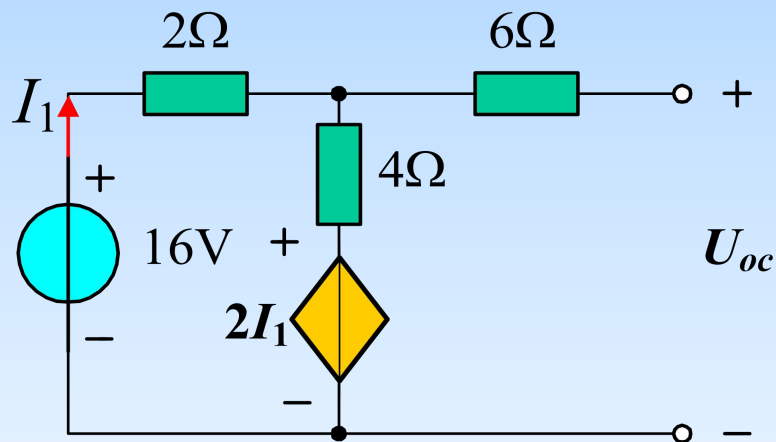


$$2I_1 + 4I_1 + 2I_1 = 16$$

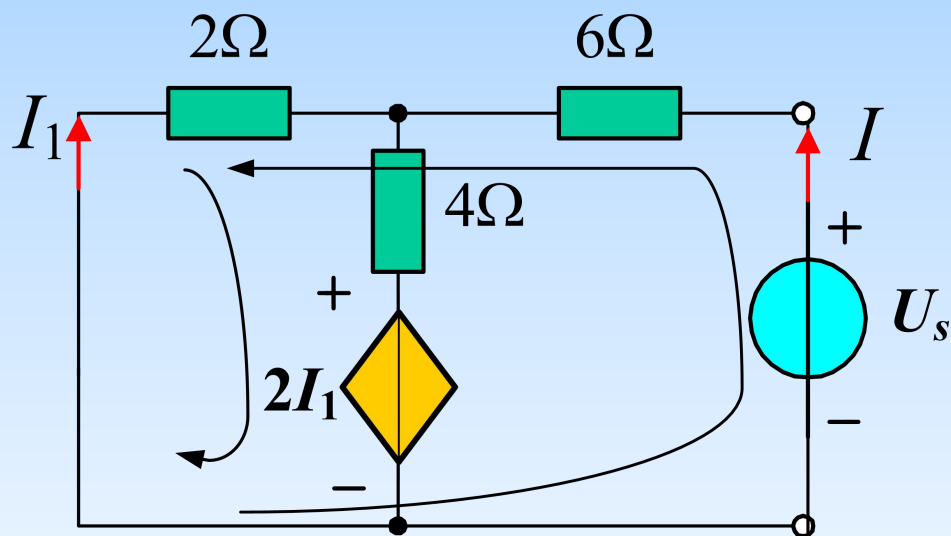
$$I_1 = 2A$$

$$U_{oc} = 4I_1 + 2I_1 = 12V$$

2、求等效电阻 R_{eq}



加压求流法 $R_{eq} = \frac{U_s}{I}$

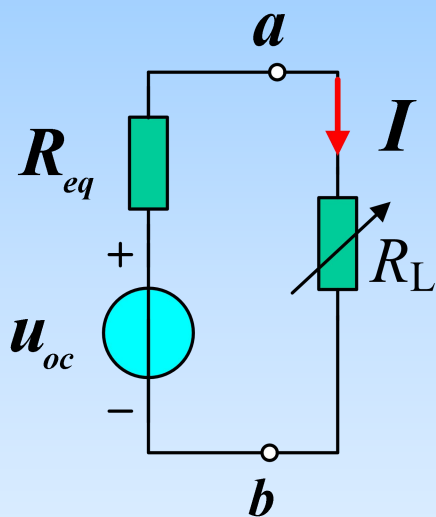


$$2I_1 + 4(I_1 + I) + 2I_1 = 0$$

$$6I - 2I_1 = U_s$$

$$I_1 = -0.5I$$

$$R_{eq} = \frac{U_s}{I} = 7\Omega$$



$$R_{eq} = 7\Omega$$

$$U_{oc} = 12V$$

当 $R_L = R_{eq}$ 时,

R_L 获得最大功率 P_{max} 。

$$P_{\max} = \frac{U_{oc}^2}{4R_{eq}} = 5.14 W$$

正弦稳态
交流电路

第五章
相量法基础

- 电感电容元件
- 复数表示及复数运算
- 正弦量
- 电路定律的相量形式
- 相量图、相量电路图

第六章
正弦稳态电路分析计算

第七章
含有耦合电感电路计算

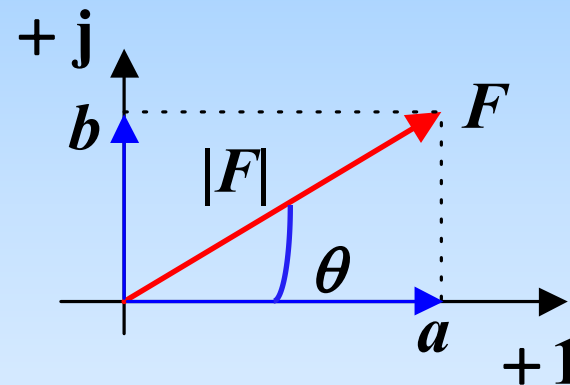
第五章 相量法基础

一、电容元件与电感元件：

	电容 C	电感 L
变量	电压 u 电荷 q	电流 i 磁链 ψ
关系式	$q = Cu$ $i = C \frac{du}{dt}$ $W_C = \frac{1}{2} Cu^2$	$\psi = Li$ $u = L \frac{di}{dt}$ $W_L = \frac{1}{2} Li^2$

二、复数表示:

$$F = a + jb = |F| (\cos \theta + j \sin \theta) \\ = |F| e^{j\theta} = |F| \angle \theta$$



$$\begin{cases} a = |F| \cos \theta \\ b = |F| \sin \theta \end{cases}$$

或

$$\begin{cases} |F| = \sqrt{a^2 + b^2} \\ \theta = \arctan \frac{b}{a} \end{cases}$$

(1) 加减运算——代数形式

(2) 乘除运算——指数形式或极坐标形式

(3) 复数相加平行四边形法

三、正弦量

正弦量的三要素

幅值、角频率、初相角

$$i(t) = I_m \cos(\omega t + \Psi_i) = \sqrt{2}I \cos(\omega t + \Psi_i)$$

$$I_m = \sqrt{2}I \quad \omega : \text{rad} \cdot \text{s}^{-1} \quad \omega = 2\pi f = \frac{2\pi}{T}$$

$$|\Psi_i| \leq \pi$$

相位差

设 $u(t) = U_m \cos(\omega t + \psi_u)$

则 相位差 φ :

$$i(t) = I_m \cos(\omega t + \psi_i)$$

$$\varphi = \psi_u - \psi_i$$

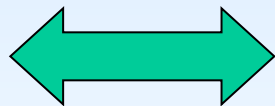
超前、滞后、同相、反相、正交

四、正弦量的相量表示

$$i(t) = \sqrt{2}I \cos(\omega t + \Psi_i) \longleftrightarrow \dot{I} = I \angle \Psi_i$$

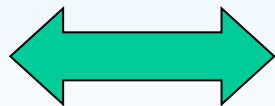
$$u(t) = \sqrt{2}U \cos(\omega t + \theta) \longleftrightarrow \dot{U} = U \angle \theta$$

正弦量
时域

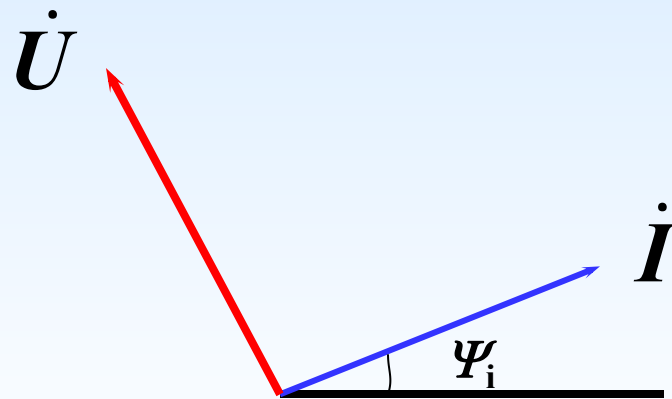


相量
频域

正弦波形图



相量图



相量图

例. $u_1(t) = 3\sqrt{2} \cos 314t \text{ V}$

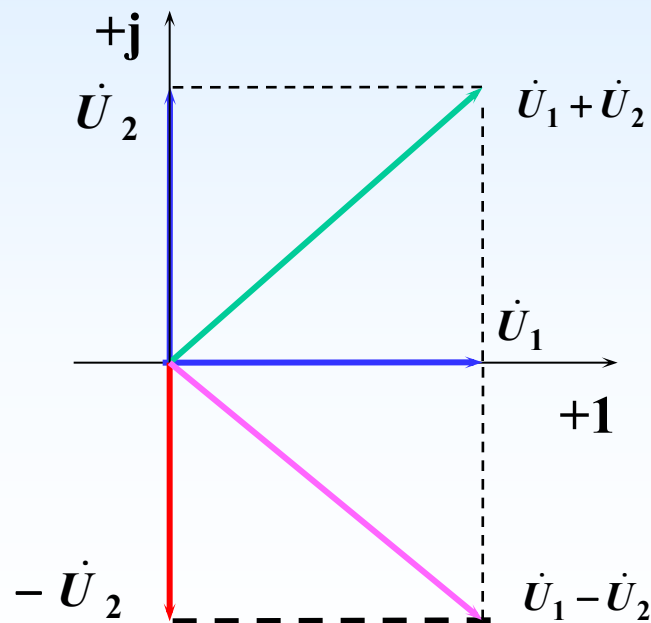
$$u_2(t) = 4\sqrt{2} \cos(314t + 90^\circ) \text{ V}$$

求: $u_1(t) + u_2(t)$

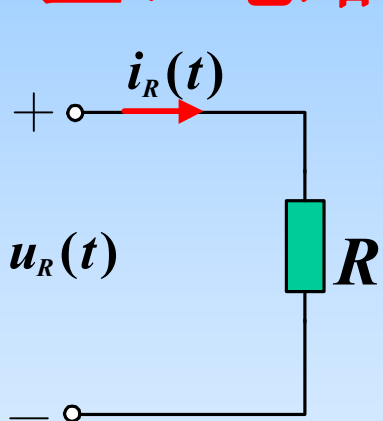
解: $\dot{U}_1 = 3 \angle 0^\circ \text{ V} \quad \dot{U}_2 = 4 \angle 90^\circ \text{ V}$

$$\dot{U} = \dot{U}_1 + \dot{U}_2 = 3 + 4 \angle 90^\circ = 5 \angle 53.1^\circ \text{ V}$$

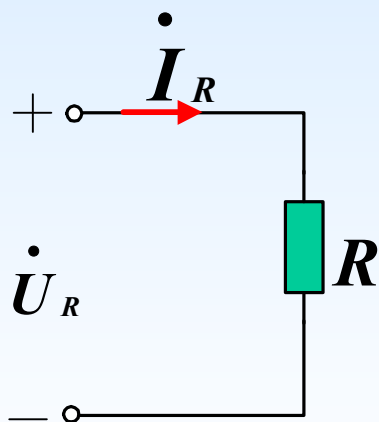
$$\begin{aligned} \therefore u(t) &= u_1(t) + u_2(t) \\ &= 5\sqrt{2} \cos(314t + 53.1^\circ) \text{ V} \end{aligned}$$



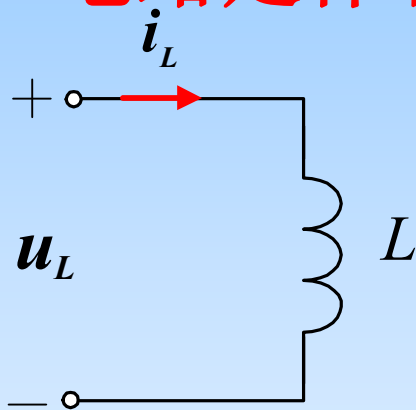
五、电路元件、电路定律相量形式



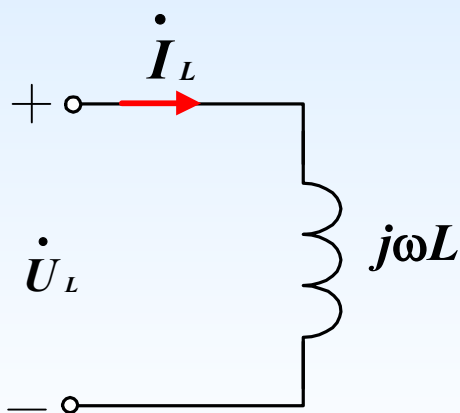
$$u_R(t) = Ri_R(t)$$



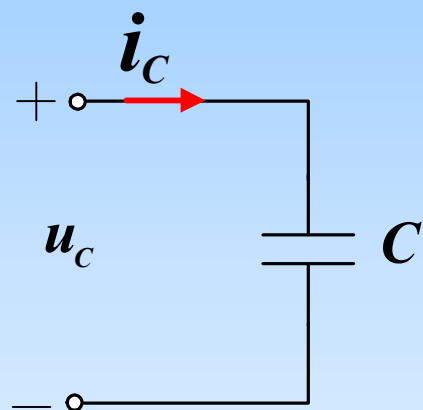
$$\dot{U}_R = R \dot{I}_R$$



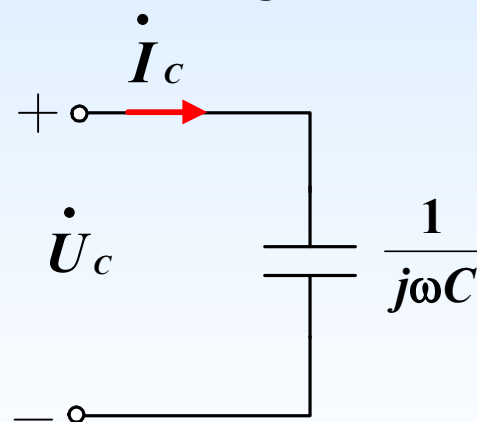
$$u_L(t) = L \frac{di_L(t)}{dt}$$



$$\dot{U}_L = j\omega L \dot{I}_L$$



$$u(t) = u(t_0) + \frac{1}{C} \int_{t_0}^t i d\xi$$



$$\dot{U}_c = \frac{1}{j\omega C} \dot{I}_c$$

电阻

电感

电容

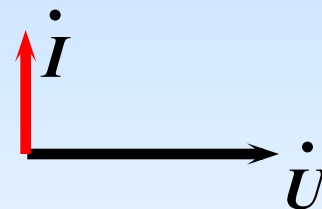
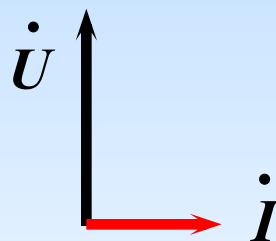
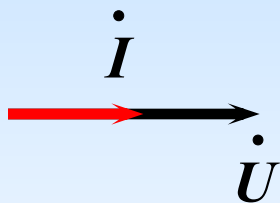
频域(相量)

$$\dot{U} = R \dot{I}$$

$$\dot{U} = j\omega L \dot{I}$$

$$\dot{U} = \frac{1}{j\omega C} \dot{I}$$

相位



有效值

$$U = RI$$

$$U = X_L I$$

$$X_L = \omega L$$

$$U = X_C I$$

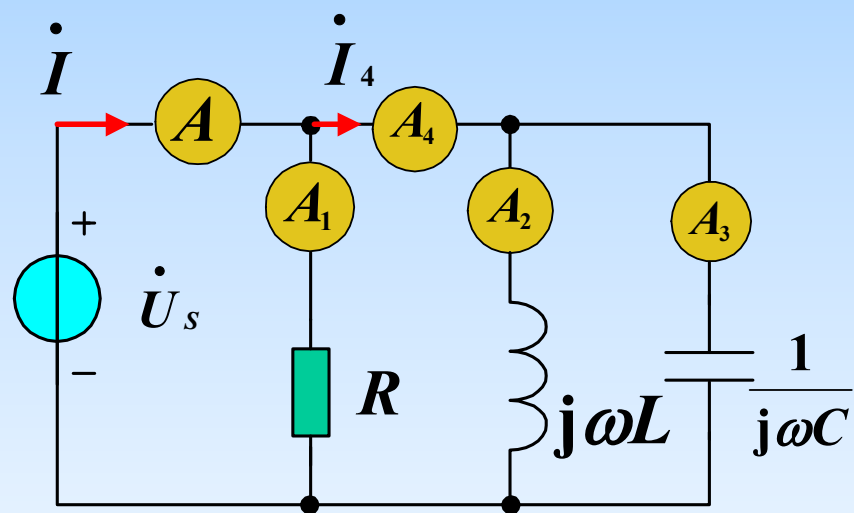
$$X_C = 1/(\omega C)$$

基尔霍夫定律的相量形式

$$\sum \dot{I} = 0$$

$$\sum \dot{U} = 0$$

例.



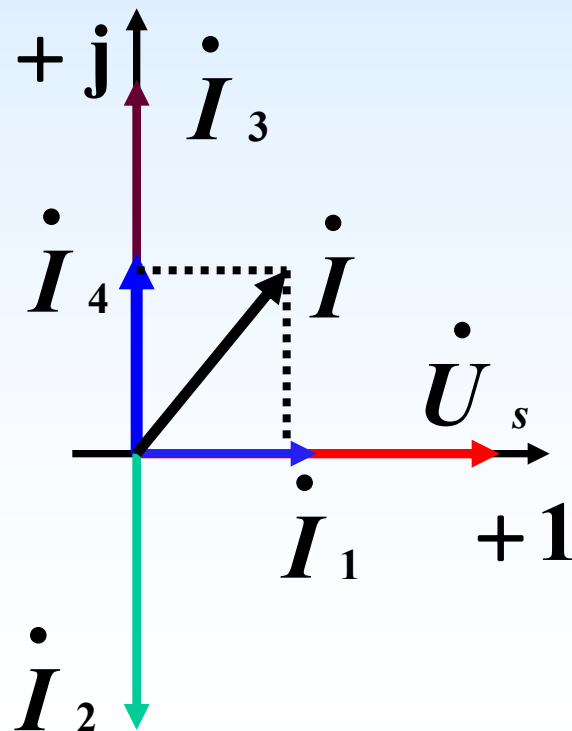
图示电路中的仪表为交流电流表，其仪表所指示的读数为电流的有效值。其中电流表A₁的读数为5A，电流表A₂的读数为20A，电流表A₃的读数为25A，求电流表A和A₄的读数。

解： 令： $\dot{U}_s = U_s \angle 0$

则： $\dot{I}_1 = 5 \angle 0^\circ \text{ A}$

$$\dot{I}_2 = 20 \angle -90^\circ \text{ A}$$

$$\dot{I}_3 = 25 \angle 90^\circ \text{ A}$$



根据KCL, 有: $\dot{I} = \dot{I}_1 + \dot{I}_2 + \dot{I}_3$

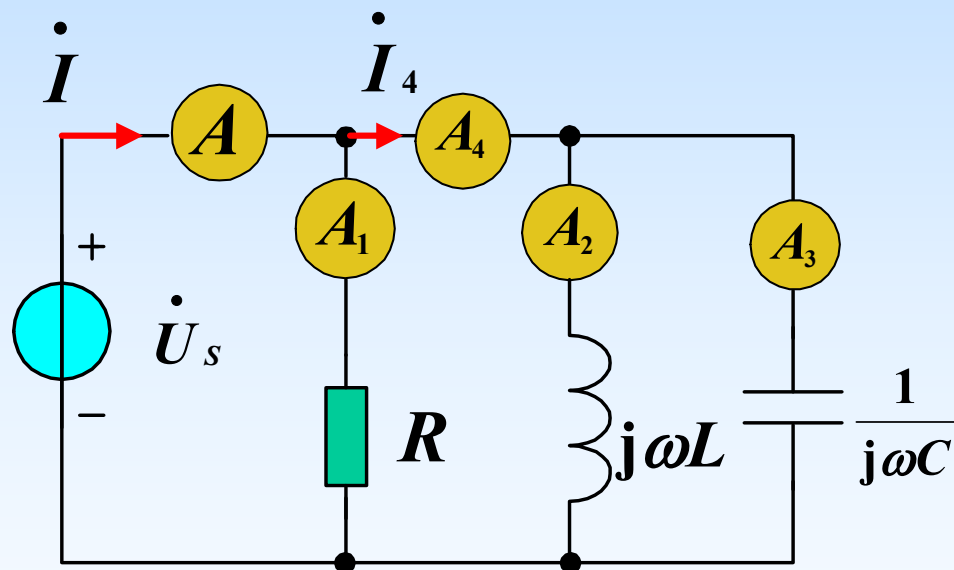
$$\dot{I} = \dot{I}_1 + \dot{I}_2 + \dot{I}_3 = 5 \angle 0^\circ + 20 \angle -90^\circ + 25 \angle 90^\circ$$

$$= 5 + (-j20) + j25$$

$$= 5 + j5$$

$$= 7.07 \angle 45^\circ \text{ A}$$

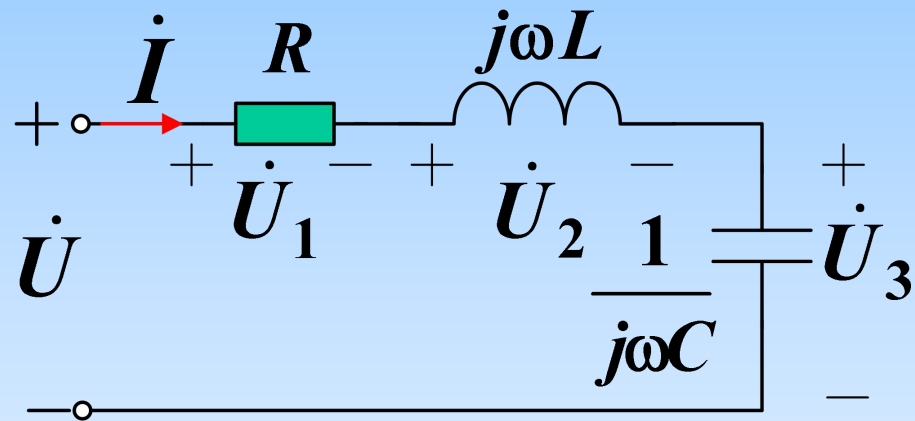
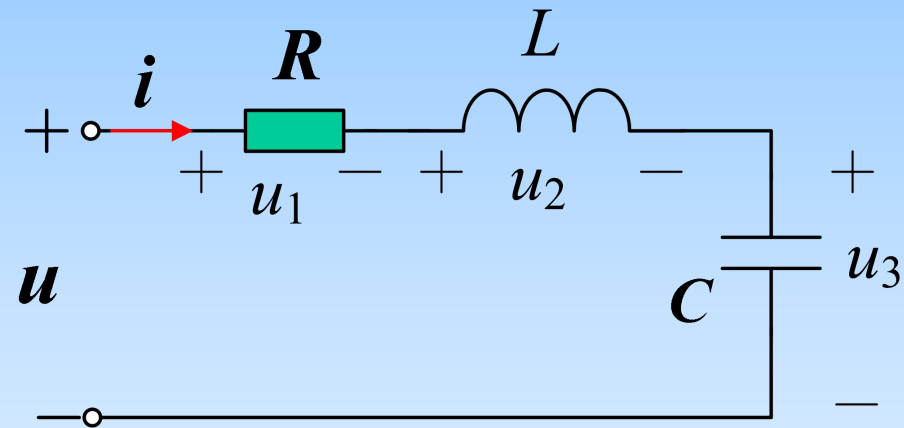
电流表A的读数为7.07A。



$$\dot{I}_4 = \dot{I}_2 + \dot{I}_3$$

$$= 20 \angle -90^\circ + 25 \angle 90^\circ = 5 \angle 90^\circ A$$

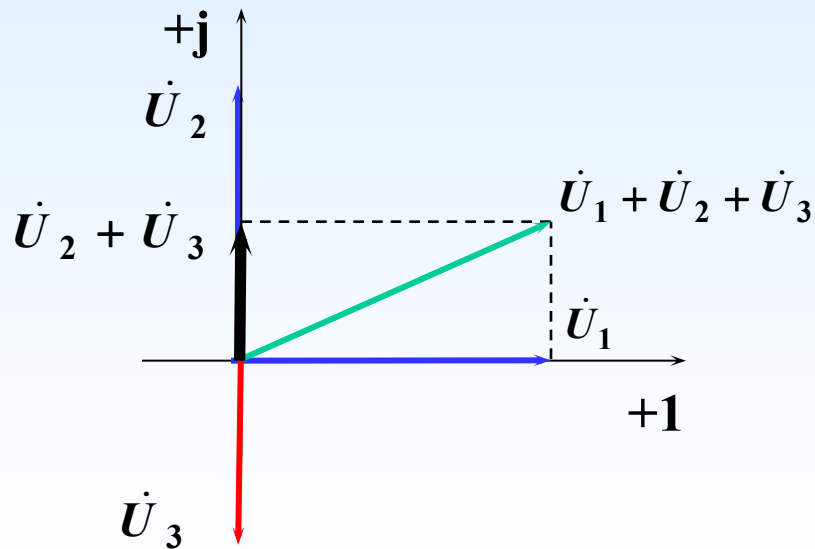
电流表A₄的读数为5A。



RLC串联电路,已知电阻电压 $U_1=15\text{V}$,电感电压 $U_2=100\text{V}$,
电容电压 $U_3=80\text{V}$,端电压 $U=?$

$$\dot{I} = I \angle 0^\circ \text{ A}$$

$$\begin{aligned} U &= \sqrt{U_1^2 + (U_2 - U_3)^2} \\ &= \sqrt{15^2 + (100^2 - 80^2)} \\ &= 25\text{V} \end{aligned}$$



某元件电压电流关联参考方向，电压电流表达式如下，请问是什么元件？

$$u = 10 \cos (10 t + 45^{\circ}) \text{ V}$$

$$i = 2 \cos (10 t + 135^{\circ}) \text{ A}$$

$$\varphi = 45^{\circ} - 135^{\circ} = -90^{\circ} \quad \text{电容元件}$$

$$Z = \frac{\dot{U}}{\dot{I}} = \frac{10 \angle 45^{\circ}}{2 \angle 135^{\circ}} = 5 \angle -90^{\circ} \Omega$$

$$\frac{1}{\omega C} = 5$$

$$C = \frac{1}{10 \times 5} = 0.02 \text{ F}$$

第五章 相量法基础

正弦稳态 交流电路

第六章 正弦稳态电路分析计算

阻抗、导纳

正弦电路计算

有功功率、无功功率、

视在功率

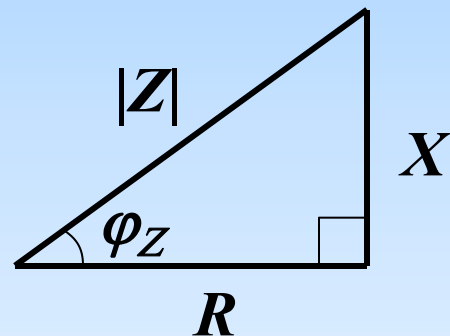
串联谐振、并联谐振

第七章 含有耦合电感电路计算

第六章 正弦稳态电路分析计算

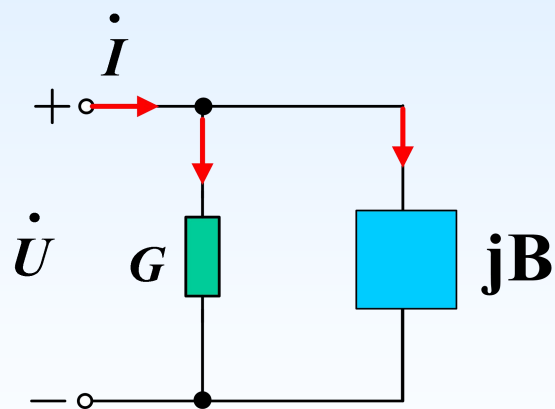
一、阻抗、导纳

$$Z = \frac{\dot{U}}{\dot{I}} = |Z| \angle \varphi_Z = R + jX$$



阻抗三角形

$$Y = \frac{\dot{I}}{\dot{U}} = |Y| \angle \varphi_Y = G + jB$$



阻抗串联、并联、混联。

已知 $u = 10 \cos 1000t V$

$i = 2 \cos(1000t - 45^\circ) A$

求阻抗，并用最简电路元件表示。

$$Z = \frac{\dot{U}}{\dot{I}} = \frac{10 \angle 0^\circ}{2 \angle -45^\circ}$$

$$= 5 \angle 45^\circ$$

$$= 5 \cos 45^\circ + j5 \sin 45^\circ$$

$$= 3.54 + j3.54 \Omega$$

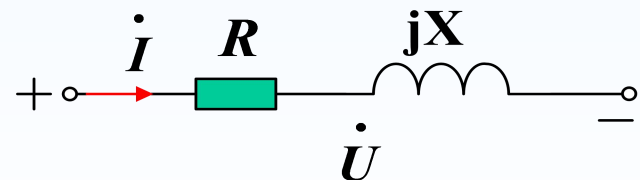
$$\dot{U}_m = 10 \angle 0^\circ V$$

$$\dot{I}_m = 2 \angle -45^\circ A$$

$$R = 3.54 \Omega$$

$$\omega L = 3.54 \Omega$$

$$L = \frac{3.54}{1000} = 3.54 mH$$



二、正弦稳态电路计算

电阻电路：

$$\left\{ \begin{array}{l} \text{KCL: } \sum i = 0 \\ \text{KVL: } \sum u = 0 \\ u = Ri \text{ 或 } i = Gu \end{array} \right.$$

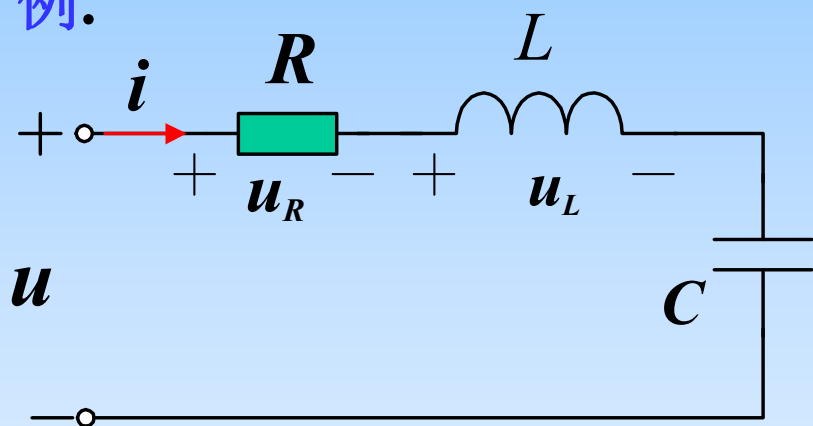
元件约束关系。

正弦电路相量分析：

$$\left\{ \begin{array}{l} \text{KCL: } \sum \dot{I} = 0 \\ \text{KVL: } \sum \dot{U} = 0 \\ \dot{U} = Z \dot{I} \text{ 或 } \dot{I} = Y \dot{U} \end{array} \right.$$

元件约束关系。

例.



已知: $R=15\Omega$, $L=0.3\text{mH}$, $C=0.2\mu\text{F}$,

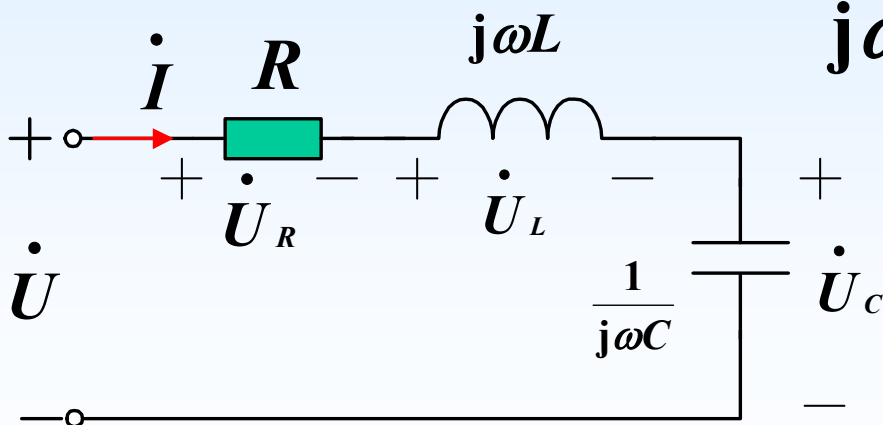
$$u = 5\sqrt{2} \cos(\omega t + 60^\circ),$$

$$f = 3 \times 10^4 \text{ Hz},$$

求 i , u_R , u_L , u_C 。

解: 其相量模型为

$$\dot{U} = 5 \angle 60^\circ \text{ V}$$



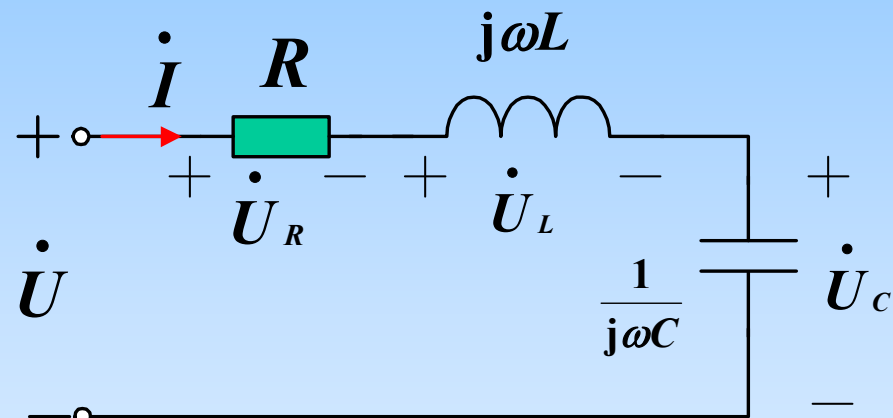
$$\begin{aligned} j\omega L &= j2\pi \times 3 \times 10^4 \times 0.3 \times 10^{-3} \\ &= j56.5\Omega \end{aligned}$$

$$\frac{1}{j\omega C} = -j \frac{1}{2\pi \times 3 \times 10^4 \times 0.2 \times 10^{-6}}$$

$$= -j26.5 \Omega$$

$$\begin{aligned}
 Z &= R + j\omega L - j\frac{1}{\omega C} \\
 &= 15 + j56.5 - j26.5 \\
 &= 33.54 \angle 63.4^\circ \Omega
 \end{aligned}$$

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{5 \angle 60^\circ}{33.54 \angle 63.4^\circ} = 0.149 \angle -3.4^\circ \text{ A}$$



$$\dot{U}_R = R \dot{I} = 15 \times 0.149 \angle -3.4^\circ = 2.235 \angle -3.4^\circ \text{ V}$$

$$\dot{U}_L = j\omega L \dot{I} = 56.5 \angle 90^\circ \times 0.149 \angle -3.4^\circ = 8.42 \angle 86.4^\circ \text{ V}$$

$$\dot{U}_C = \frac{1}{j\omega C} \dot{I} = 26.5 \angle -90^\circ \times 0.149 \angle -3.4^\circ = 3.95 \angle -93.4^\circ \text{ V}$$

则

$$i = 0.149\sqrt{2} \cos(\omega t - 3.4^\circ) \text{ A}$$

$$u_R = 2.235\sqrt{2} \cos(\omega t - 3.4^\circ) \text{ V}$$

$$u_L = 8.42\sqrt{2} \cos(\omega t + 86.6^\circ) \text{ V}$$

$$u_C = 3.95\sqrt{2} \cos(\omega t - 93.4^\circ) \text{ V}$$

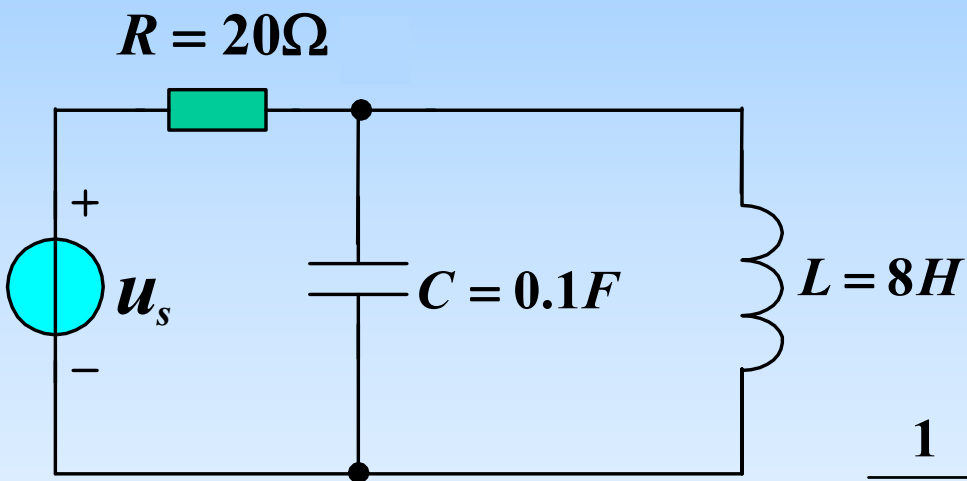
$$\dot{I} = 0.149 \angle -3.4^\circ \text{ A}$$

$$\dot{U}_R = 2.235 \angle -3.4^\circ \text{ V}$$

$$\dot{U}_L = 8.42 \angle 86.4^\circ \text{ V}$$

$$\dot{U}_C = 3.95 \angle -93.4^\circ \text{ V}$$

用求各支路电流。

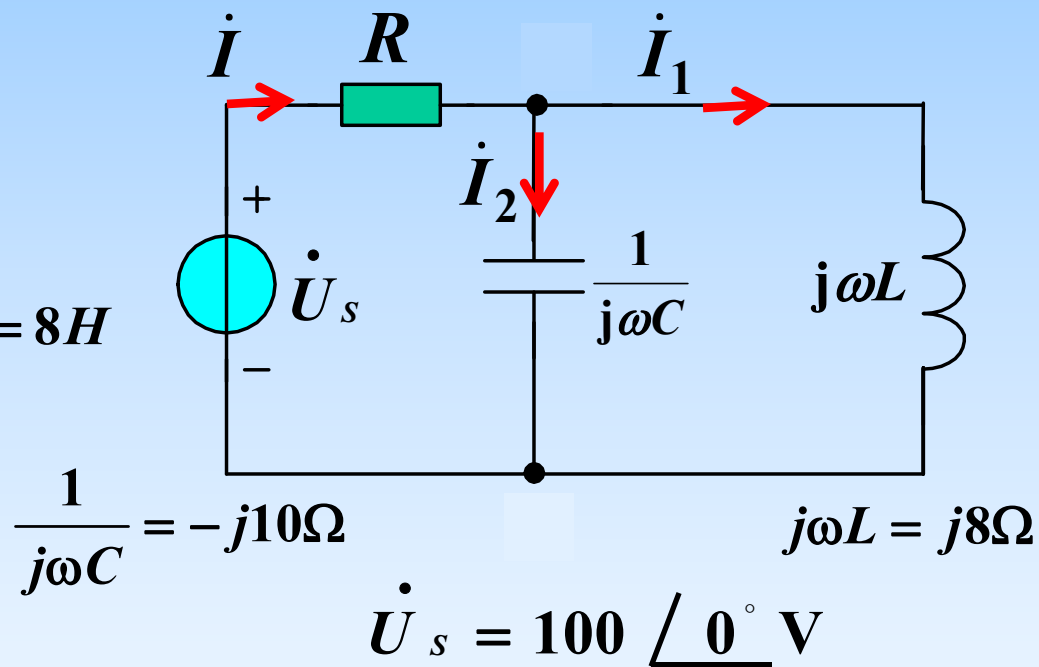


$$u_s = 100 \sqrt{2} \cos t \text{ V}$$

$$Z = 20 + \frac{-j10 \times j8}{-j10 + j8} = 20 + j40\Omega$$

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{100\angle 0^\circ}{20 + j40} = \frac{100\angle 0^\circ}{44.72\angle 63.43^\circ} = 2.23\angle -63.5^\circ A$$

$$\dot{I}_2 = \frac{j8}{-j10 + j8} \dot{I} = -8.95\angle -63.5^\circ = 8.95\angle 116.5^\circ A$$

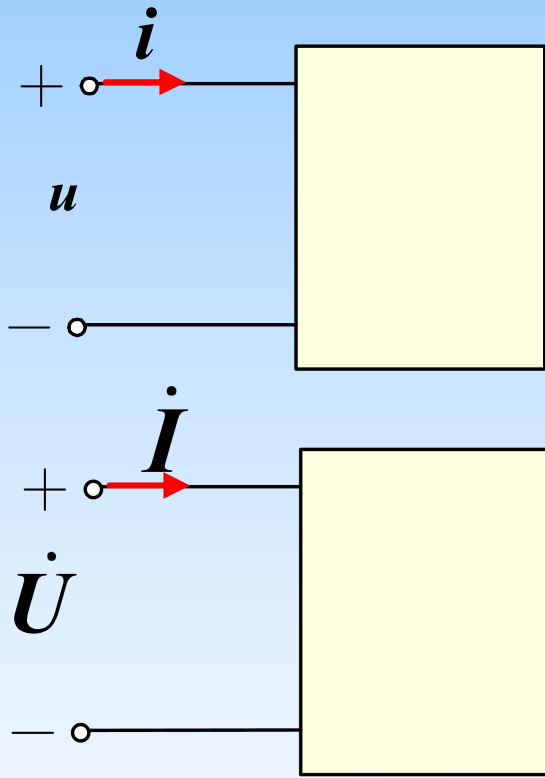


$$\dot{I}_1 = \frac{-j10}{-j10 + j8} \dot{I} = 11.21\angle -63.5^\circ A$$

$$\dot{I}_1 = 11.21 \angle -63.5^\circ \text{ A} \quad \text{则} \quad \dot{i}_1 = 11.21 \sqrt{2} \cos(t - 63.5^\circ) \text{ A}$$

$$\dot{I}_2 = 8.95 \angle 116.5^\circ \text{ A} \quad i_2 = 8.95 \sqrt{2} \cos(t + 116.5^\circ) \text{ A}$$

$$\dot{I} = 2.23 \angle -63.5^\circ \text{ A} \quad i = 2.23 \sqrt{2} \cos(t - 63.5^\circ) \text{ A}$$



三、有功功率、无功功率、视在功率

$$u(t) = \sqrt{2}U \cos(\omega t + \psi_u)$$

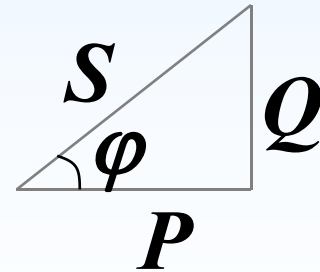
$$i(t) = \sqrt{2}I \cos(\omega t + \psi_i)$$

$$\varphi = \psi_u - \psi_i$$

$$P = UI \cos \varphi \quad \text{单位: W}$$

$$Q = UI \sin \varphi \quad \text{单位: var}$$

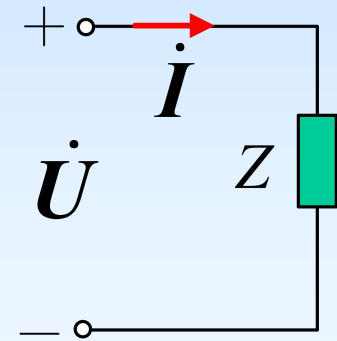
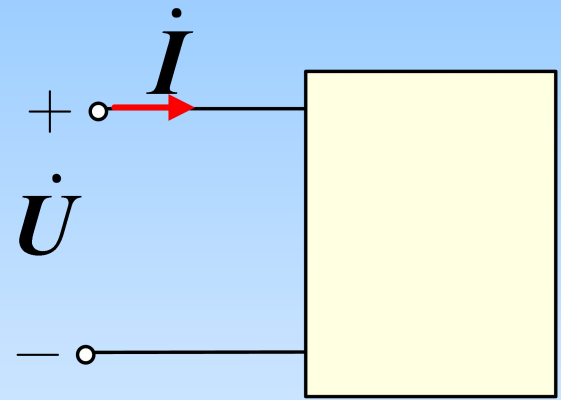
$$S = UI \quad \text{单位: VA}$$

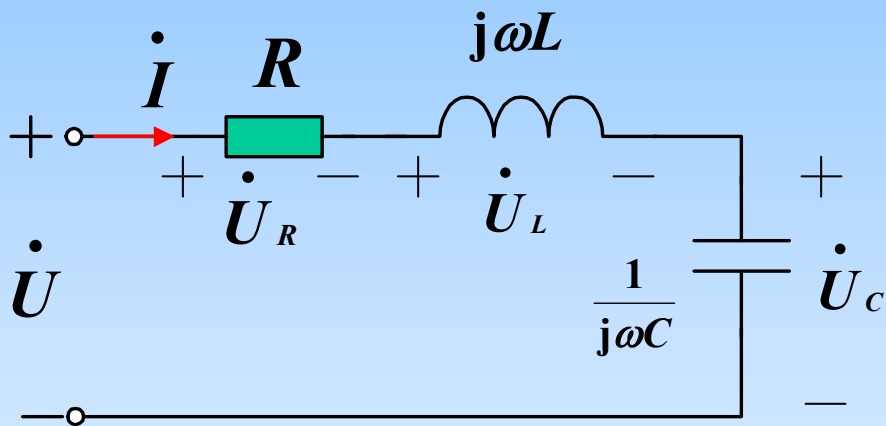


$$Z = R + \mathrm{j}X = |Z| \angle \varphi_z$$

$$P = RI^2 \quad \text{吸收有功功率}$$

$$Q = XI^2 \quad \text{吸收无功功率}$$





$$P = UI \cos \varphi$$

$$\dot{U} = 5 \angle 60^\circ \text{ V}$$

$$= 5 \times 0.149 \times \cos(60^\circ - (-3.4^\circ)) = 0.33 \text{ W}$$

$$\dot{I} = 0.149 \angle -3.4^\circ \text{ A}$$

$$Q = UI \sin \varphi$$

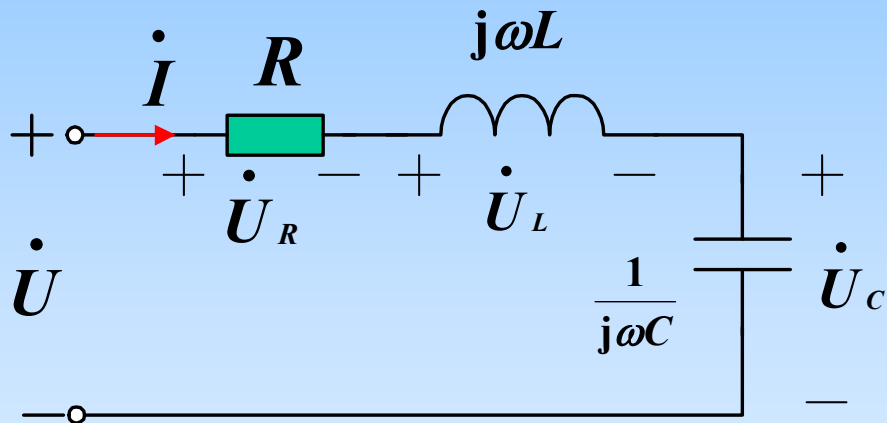
$$\dot{U}_R = 2.235 \angle -3.4^\circ \text{ V} \quad = 5 \times 0.149 \times \sin(60^\circ - (-3.4^\circ)) = 0.66 \text{ Var}$$

$$\dot{U}_L = 8.42 \angle 86.4^\circ \text{ V}$$

$$S = UI$$

$$\dot{U}_C = 3.95 \angle -93.4^\circ \text{ V}$$

$$= 5 \times 0.149 = 0.745 \text{ VA}$$



$$\dot{I} = 0.149 \angle -3.4^\circ \text{ A}$$

$$Z = R + j\omega L - j\frac{1}{\omega C}$$

$$= 15 + j56.5 - j26.5$$

$$= 15 + j30 \Omega$$

$$P = RI^2 = 15 \times 0.149^2 = 0.33 \text{ W}$$

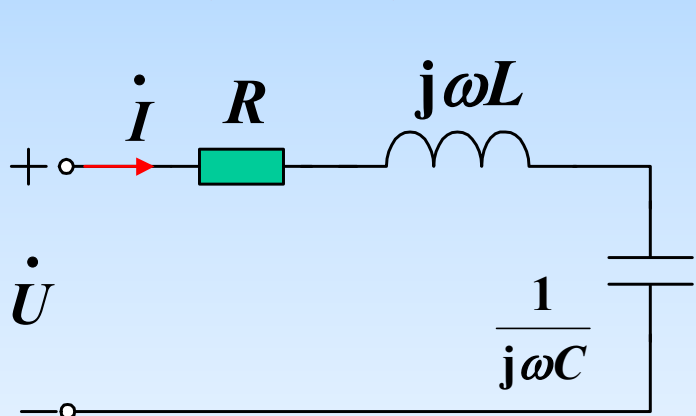
吸收有功功率

$$Q = XI^2 = 30 \times 0.149^2 = 0.66 \text{ Var}$$

吸收无功功率

四、串联谐振、并联谐振

RLC串联谐振



$$Z = R + j(\omega L - \frac{1}{\omega C}) = |Z| \angle \varphi$$

$$\omega L < \frac{1}{\omega C} \quad \text{容性}$$

$$\omega L > \frac{1}{\omega C} \quad \text{感性}$$

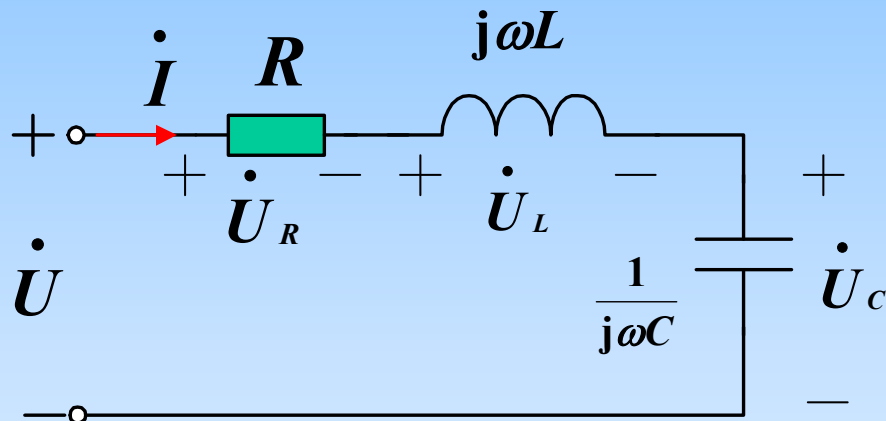
$$\omega L = \frac{1}{\omega C} \quad \text{电阻性} \quad X = 0$$

谐振: $Z = R$

当满足一定条件(对RLC串联电路, 使 $(\omega L = 1/\omega C)$, 电路中电压、电流同相, 电路的这种状态称为谐振。

谐振条件

$$\omega = \omega_0 = \frac{1}{\sqrt{LC}}$$



U 一定时电流 I 达到最大值。

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$I = \frac{U}{|Z|} = \frac{U}{R}$$

电阻吸收的有功功率 P 是最大值。

电阻上的电压等于电源电压， LC 上串联总电压为零。

$$\dot{U}_R = R \dot{I} = \dot{U}$$

$$U_L = QU$$

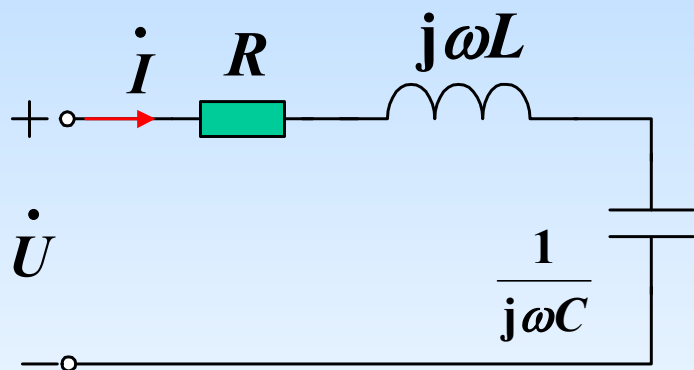
$$\dot{U}_L + \dot{U}_C = 0$$

$$U_C = QU$$

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

无量纲

例： 已知： $L=20\text{mH}$ ， $C=200\text{pF}$ ， $R=100\ \Omega$ ， 正弦电压源电压 $U=10\text{V}$ ， 求电路的谐振频率 f_0 电路的 Q 值即谐振时 U_L 、 U_C 。



解：

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

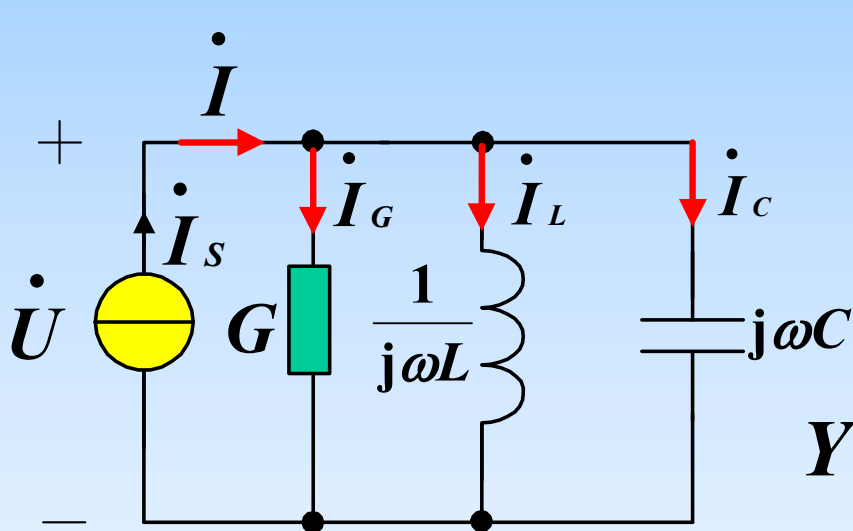
$$= \frac{1}{2\pi\sqrt{20 \times 10^{-3} \times 200 \times 10^{-12}}}$$

$$= 79.6\text{KHz}$$

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{100} \sqrt{\frac{20 \times 10^{-3}}{200 \times 10^{-12}}} = 100$$

$$U_L = U_C = QU = 100 \times 10 = 1000\text{V}$$

二、RLC并联谐振



$$Y = G + j(\omega C - \frac{1}{\omega L})$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Y(j\omega_0) = G + j(\omega_0 C - \frac{1}{\omega_0 L}) \\ = G$$

并联谐振

并联谐振定义与串联谐振电路的定义相同，即端口上的电压与电流 同相时的工作状况称为谐振。

GLC并联电路谐振时的特点

$$Y(j\omega_0) = G + j(\omega_0 C - \frac{1}{\omega_0 L}) \\ = G$$

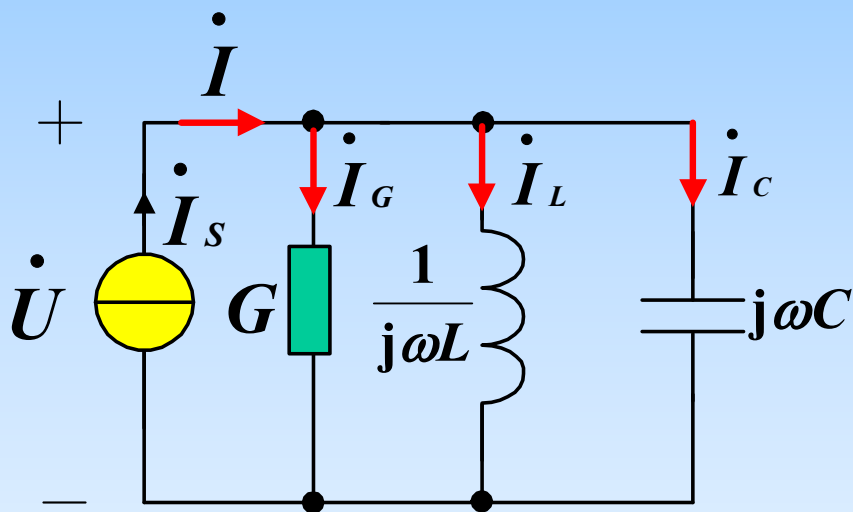
输入阻抗 $Z(j\omega_0) = R$ 为最大。

谐振时端电压 U 达到最大值。

电阻上的电流等于电流源的电流。

$$\dot{I}_R = G\dot{U} = \dot{I}_S \quad \dot{I}_L + \dot{I}_C = 0$$

并联谐振又称电流谐振。



第五章
相量法基础

第六章
正弦稳态电路分析计算

第七章
含有耦合电感电路计算

基本概念

电压电流关系

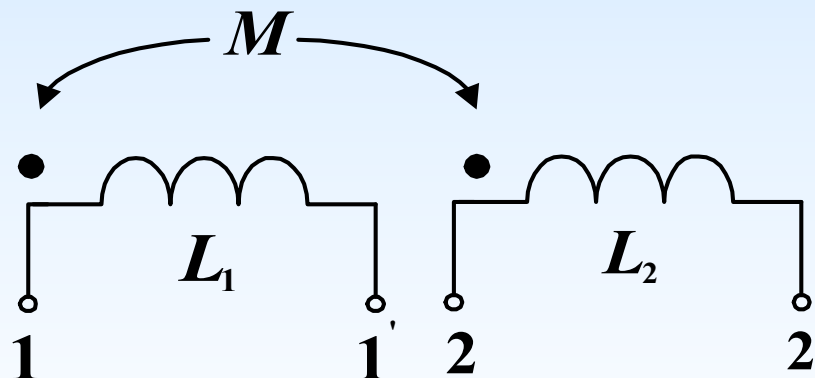
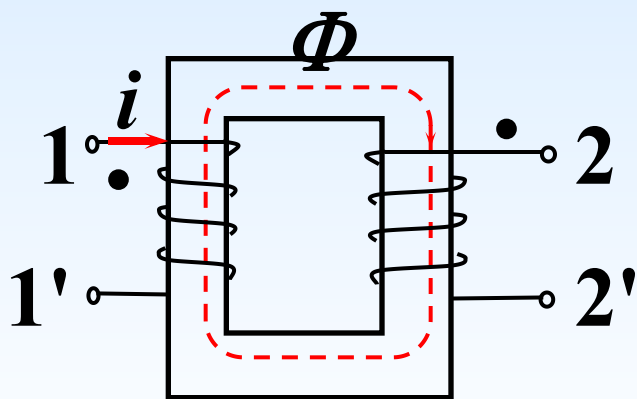
含有耦合电感电路计算

正弦稳态
交流电路

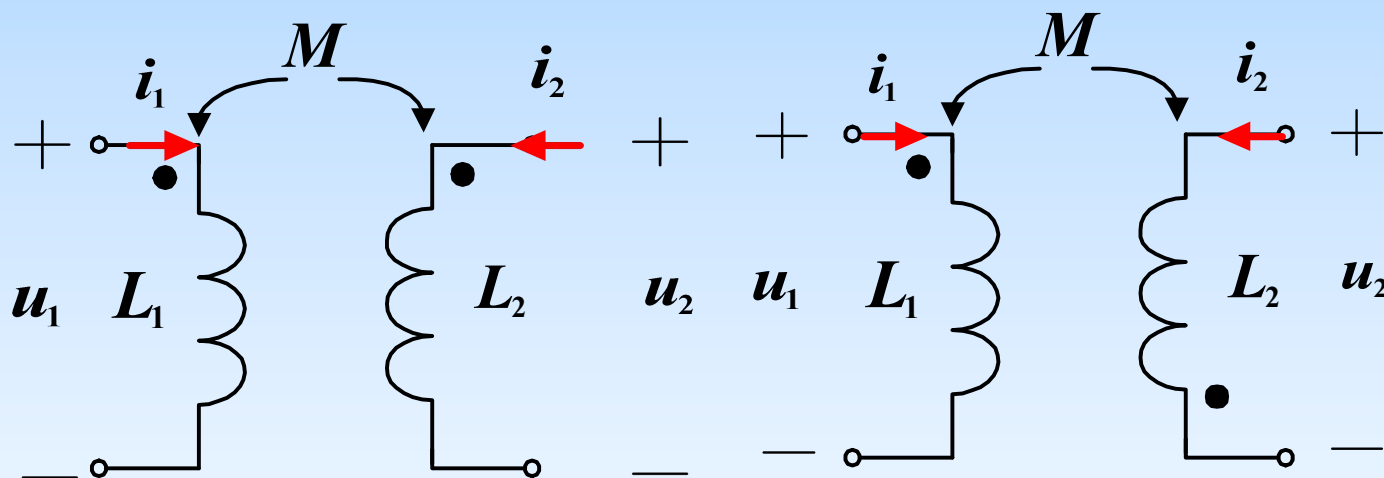
第八章 含有耦合电感电路计算

基本概念

1、同名端：当两个电流分别从两个线圈的对应端子流入，其所产生的磁场相互加强时，则这两个对应端子称为同名端。同名端用“*”或“●”表示。

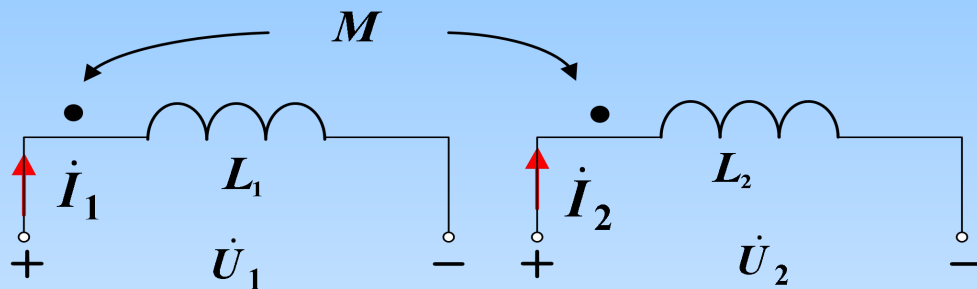


2、电压电流关系

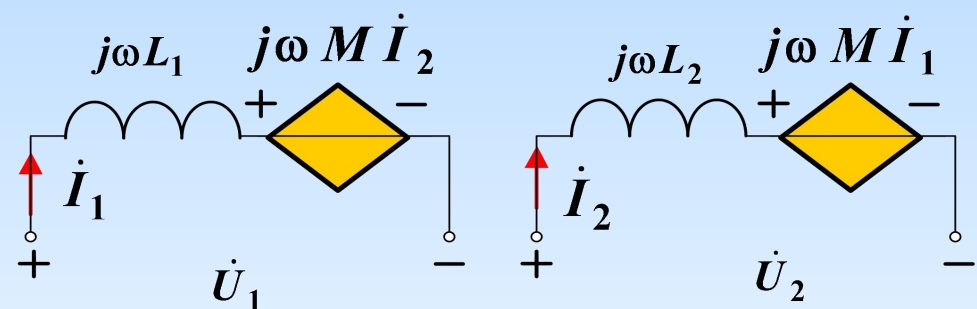


时域形式:
$$u_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$
$$u_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$u_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$
$$u_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

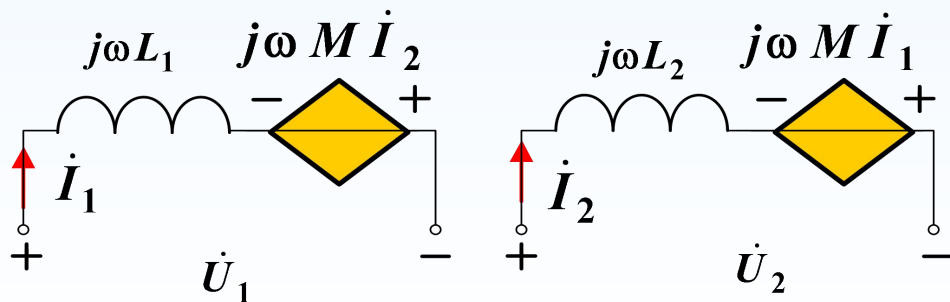
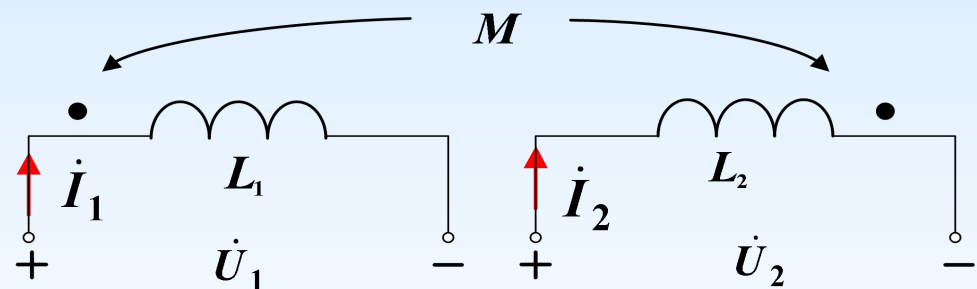


$Z_M = j\omega M$ 互感阻抗。



$$\dot{U}_1 = j\omega L_1 \dot{I}_1 + j\omega M \dot{I}_2$$

$$\dot{U}_2 = +j\omega M \dot{I}_1 + j\omega L_2 \dot{I}_2$$

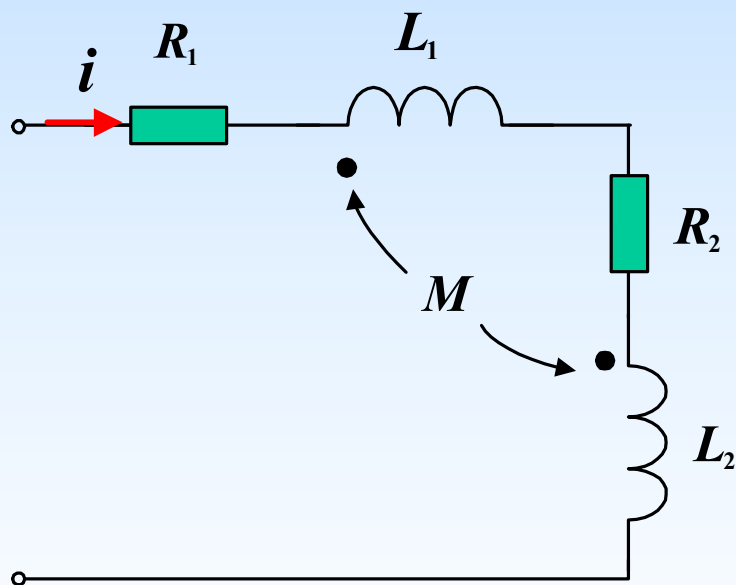


$$\dot{U}_1 = j\omega L_1 \dot{I}_1 - j\omega M \dot{I}_2$$

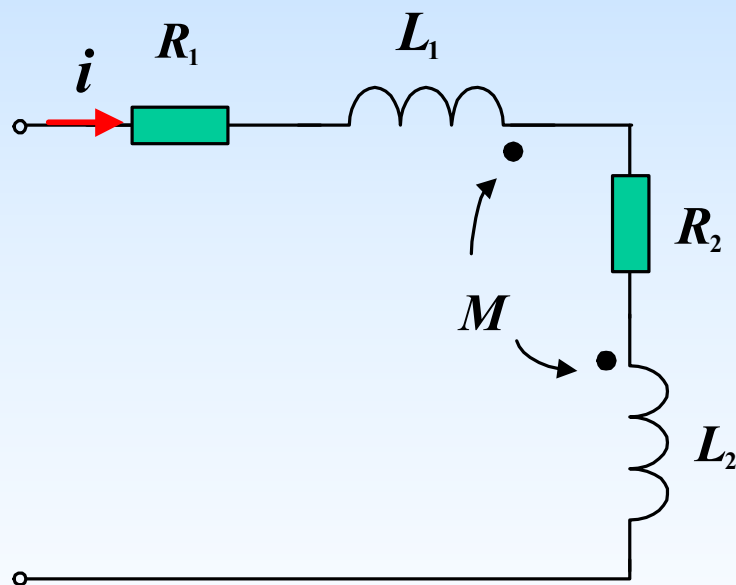
$$\dot{U}_2 = -j\omega M \dot{I}_1 + j\omega L_2 \dot{I}_2$$

含有耦合电感电路的计算

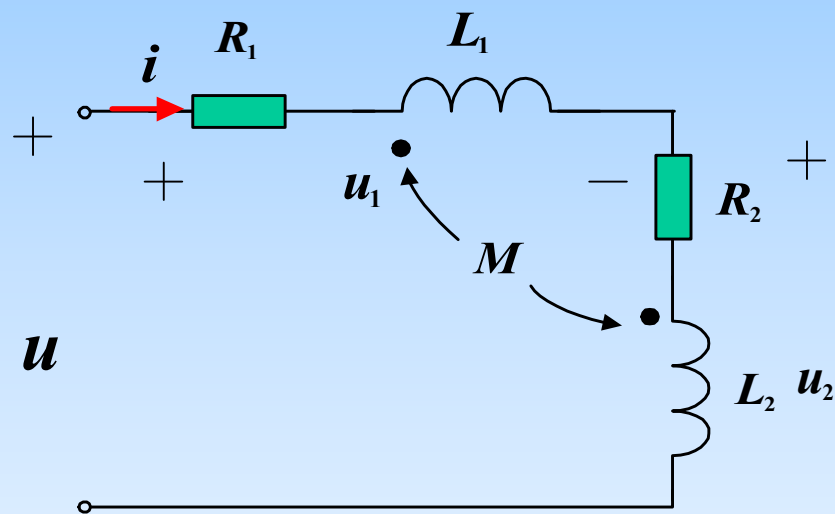
一、互感线圈的串联



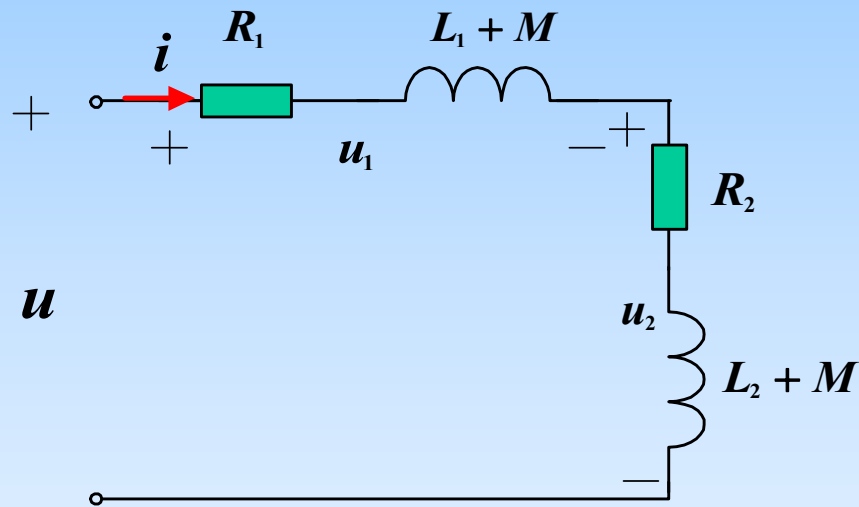
顺接



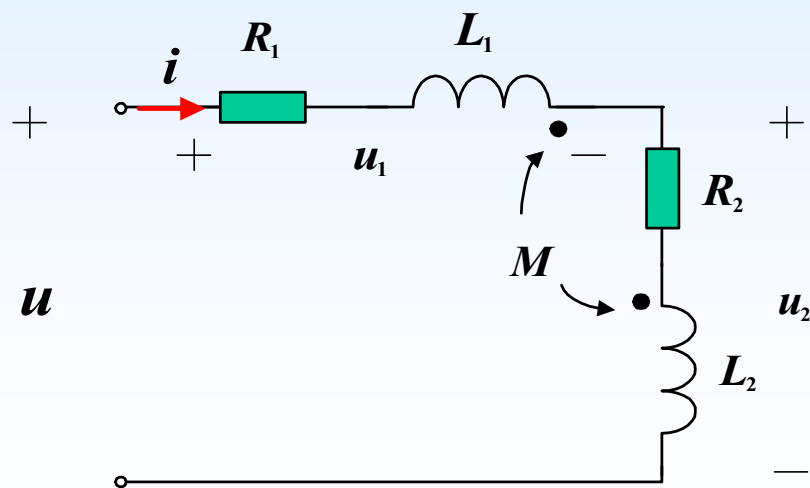
反接



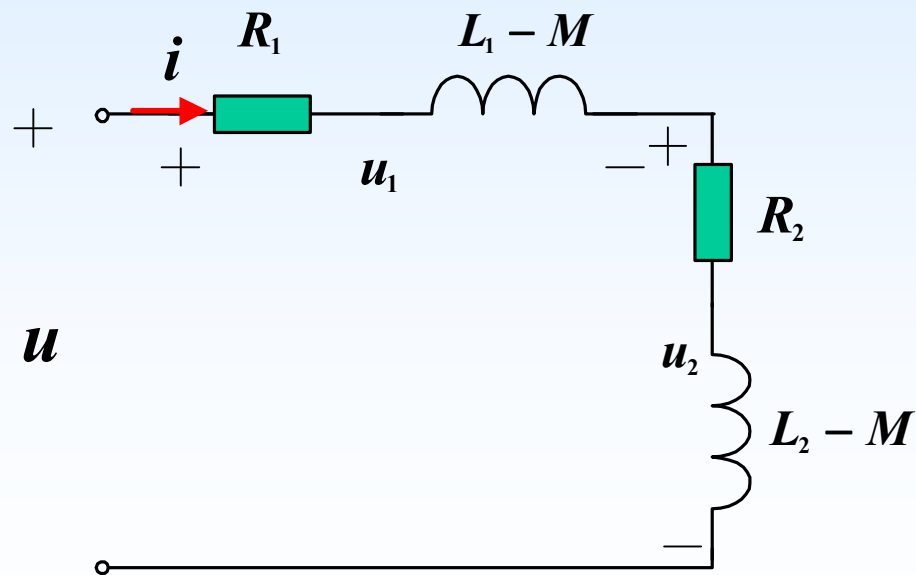
顺接



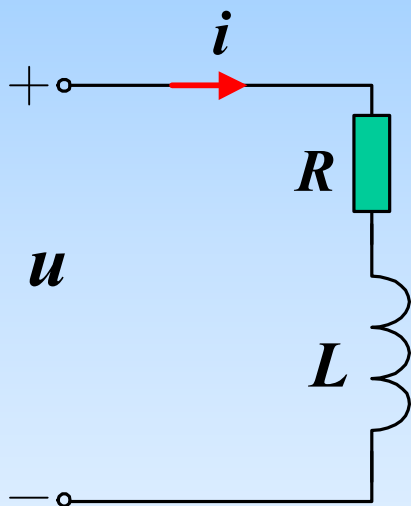
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反接



去耦等效电路

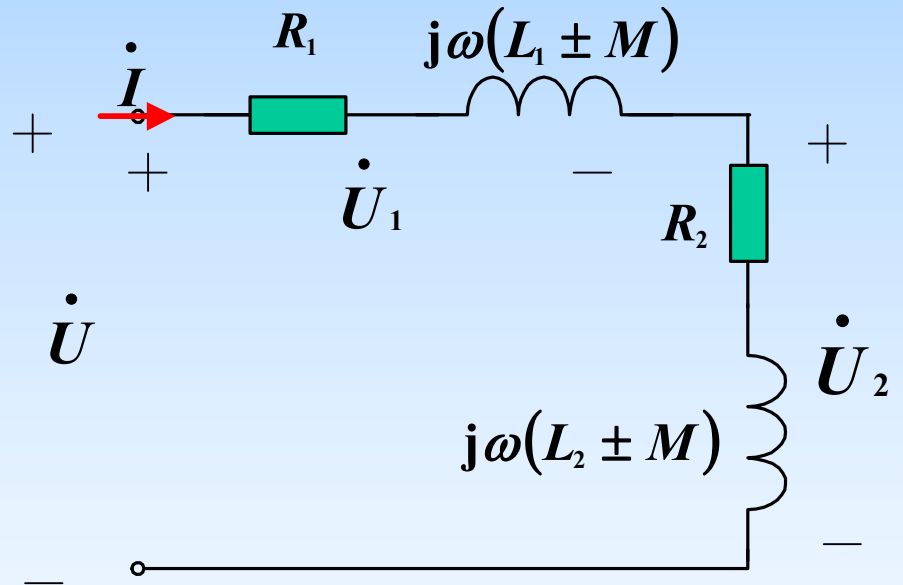
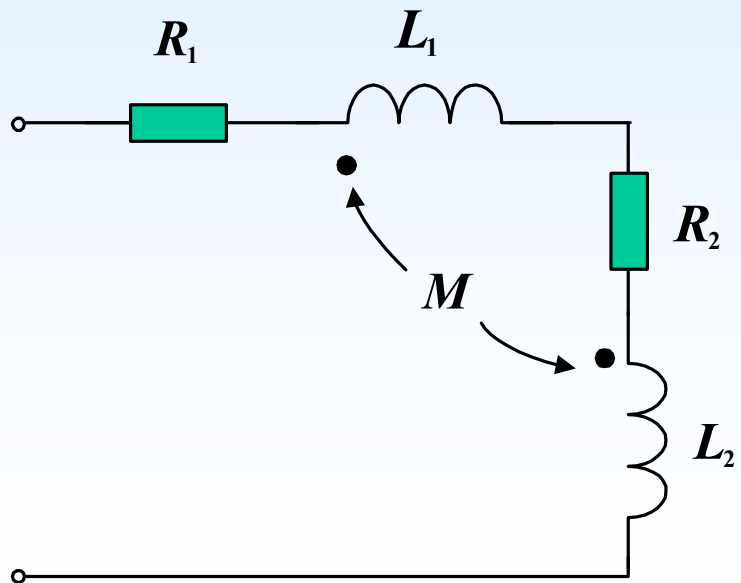
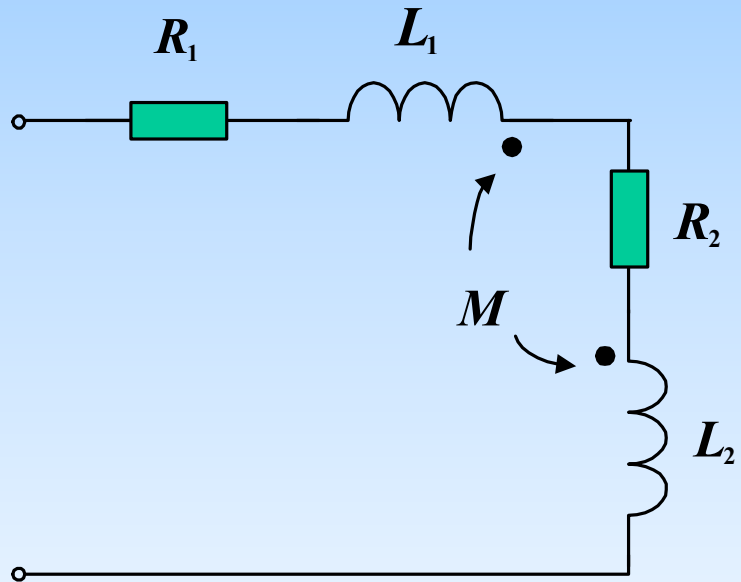


$$\therefore R = R_1 + R_2$$

$$L = L_1 + L_2 \pm 2M$$

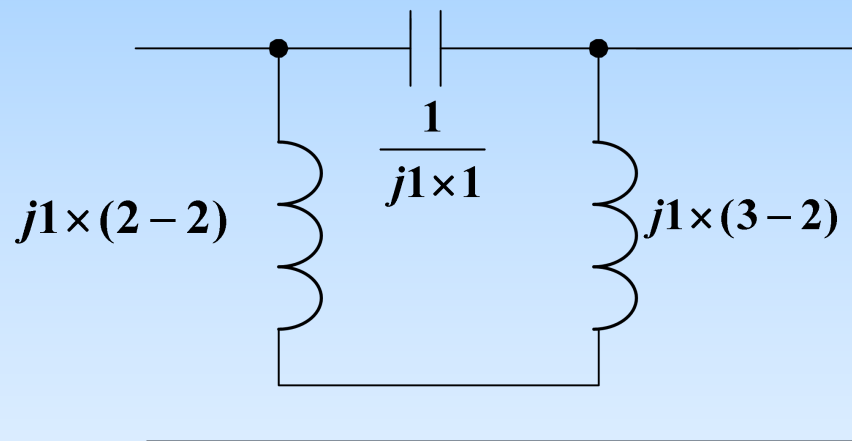
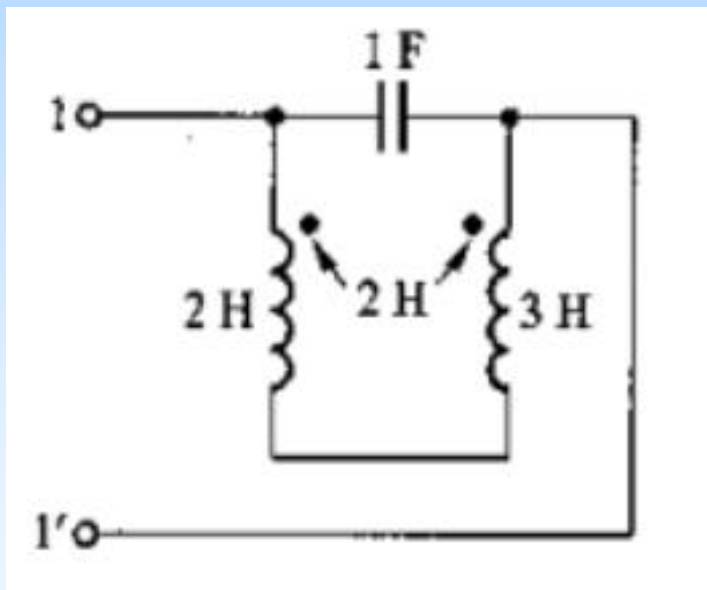
顺接时等效电感增加，反接时等效电感减少。

“+”顺接 “-”反接



去耦等效电路

求输入阻抗。 $\omega=1\text{rad/s}$

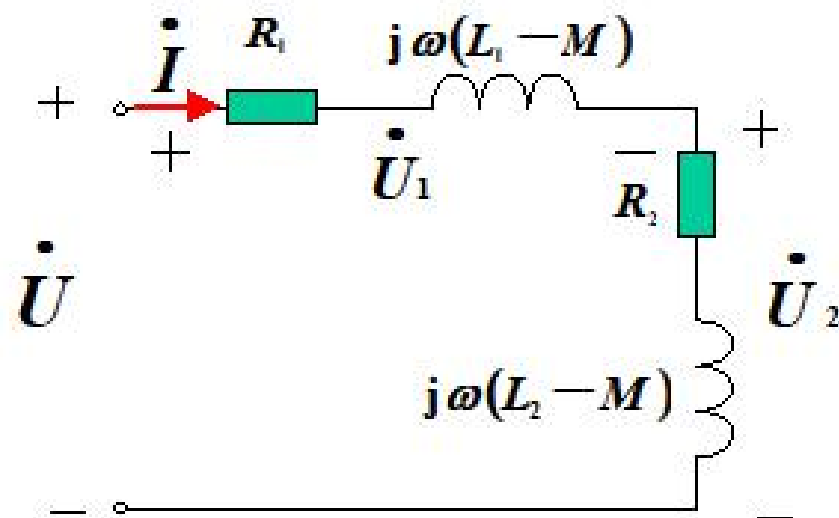
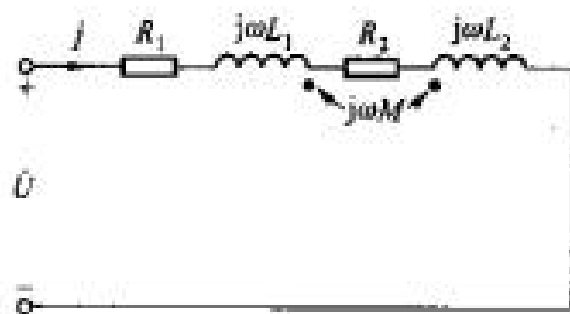


$$Z = \infty$$

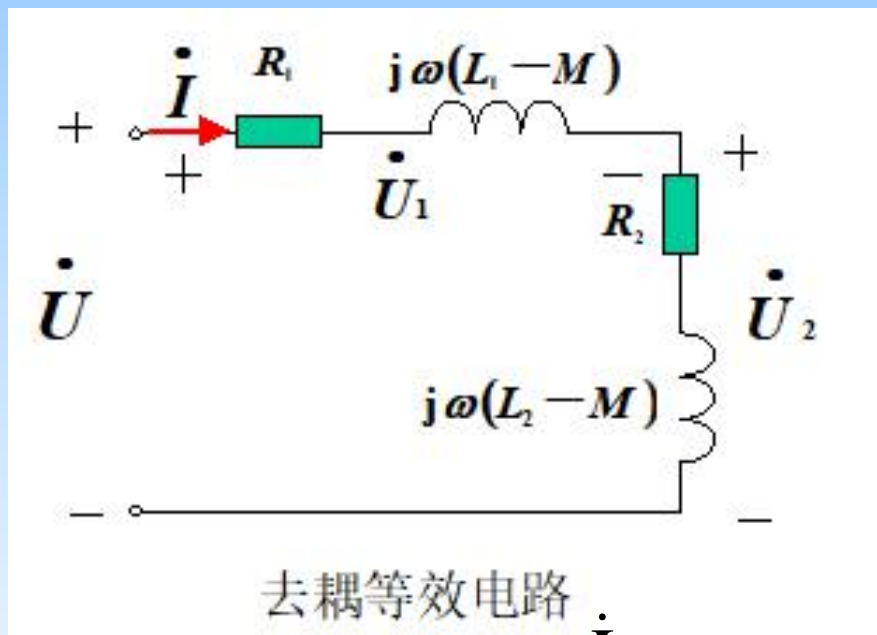
$$Y = 0$$

10-8 电路如题 10-8 图所示, 已知两个线圈的参数为: $R_1 = R_2 = 100\ \Omega$, $L_1 = 3\ \text{H}$, $L_2 = 10\ \text{H}$, $M = 5\ \text{H}$, 正弦电源的电压 $U = 220\ \text{V}$, $\omega = 100\ \text{rad/s}$ 。

- (1) 试求两个线圈端电压, 并作出电路的相量图;
- (2) 证明两个耦合电感反向串联时不可能有 $L_1 + L_2 - 2M \leq 0$;
- (3) 电路中串联多大的电容可使 $\dot{U}$ 、 i 同相?
- (4) 画出该电路的去耦等效电路。



去耦等效电路



$$R_1=R_2=100\Omega, U=220V, \omega=100$$

$$L_1=3H, L_2=10H, M=5H.$$

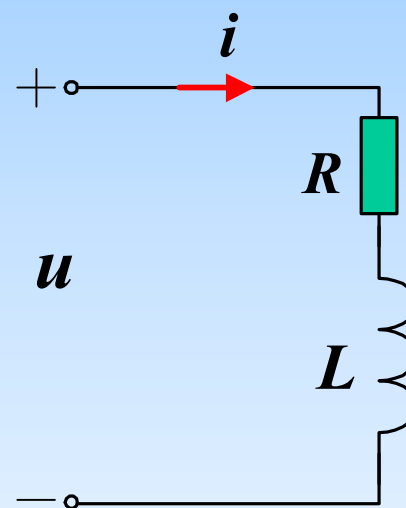
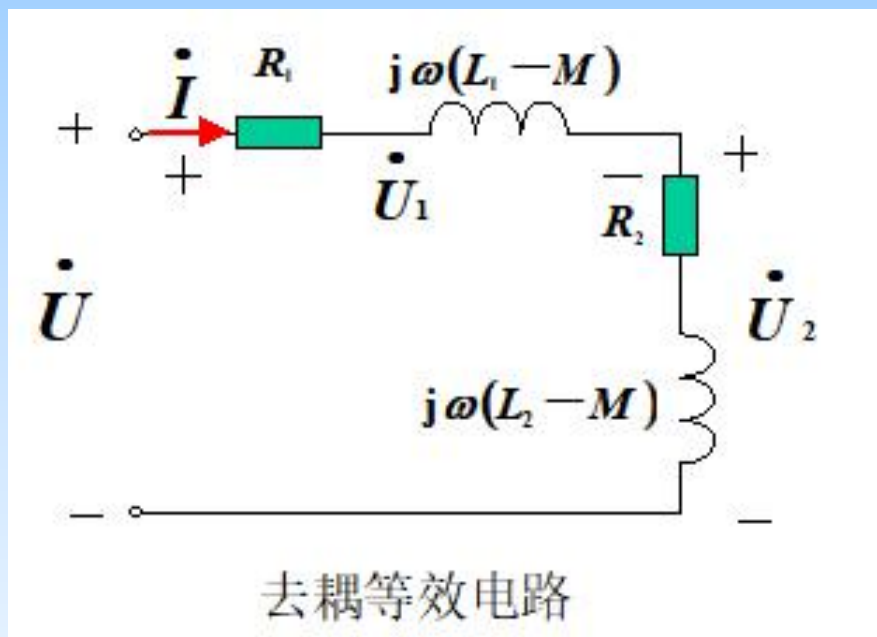
$$\text{设 } \dot{U} = 220\angle 0^\circ V$$

$$\dot{I} = \frac{\dot{U}}{R_1 + j\omega(L_1 - M) + R_2 + j\omega(L_2 - M)}$$

$$= \frac{220 \angle 0^\circ}{200 + j300} = 0.61 \angle -56.31^\circ A$$

线圈1 $\dot{U}_1 = (R_1 + j\omega(L_1 - M)) \times \dot{I} = (100 - j200) \times 0.61 \angle -56.31^\circ = 136.4 \angle -119.74^\circ V$

线圈2 $\dot{U}_2 = (R_2 + j\omega(L_2 - M)) \times \dot{I} = (100 + j500) \times 0.61 \angle -56.31^\circ = 311.04 \angle 22.38^\circ V$



$$R = 200\Omega$$

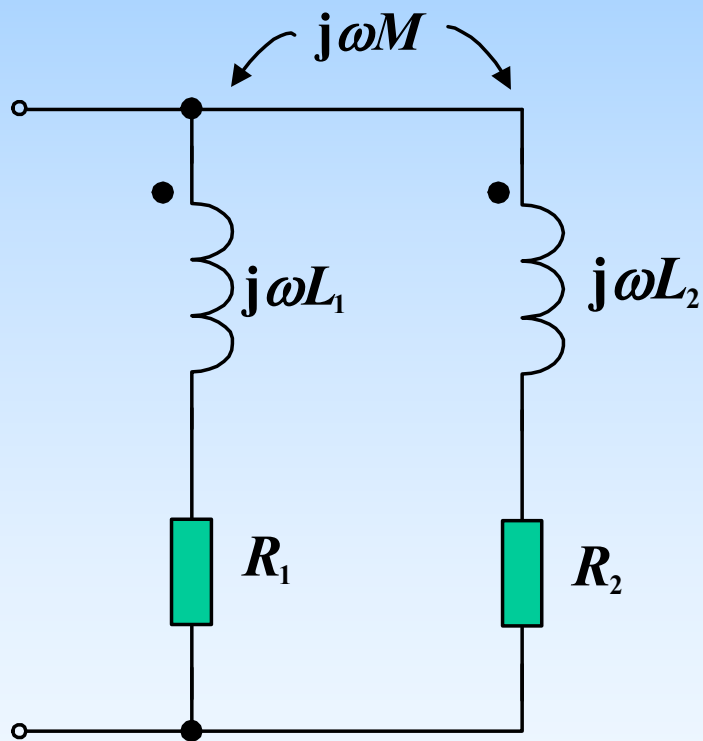
$$L = L_1 + L_2 - 2M = 3H$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

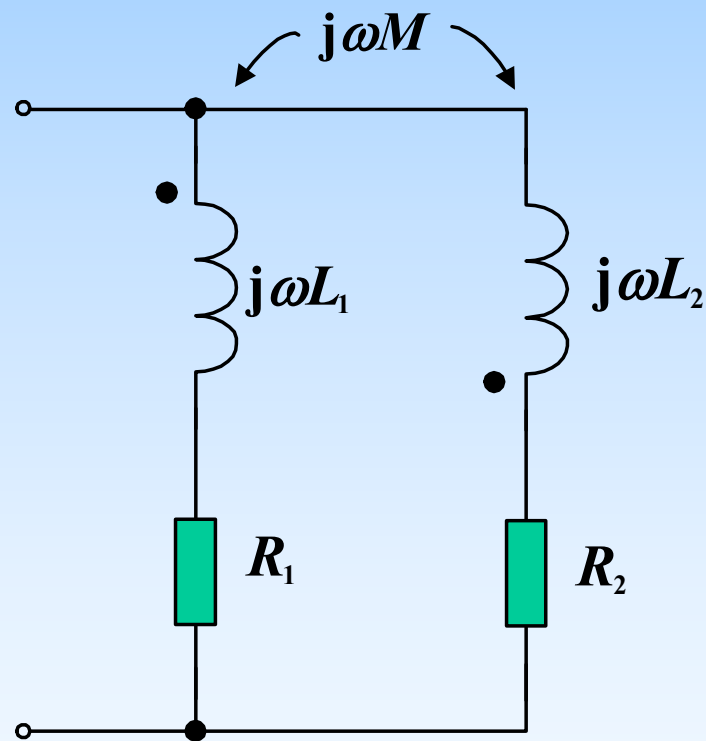
$$100 = \frac{1}{\sqrt{3C}}$$

$$C = 33.3\mu F$$

二、互感线圈的同侧、异侧连接



同侧连接



异侧连接

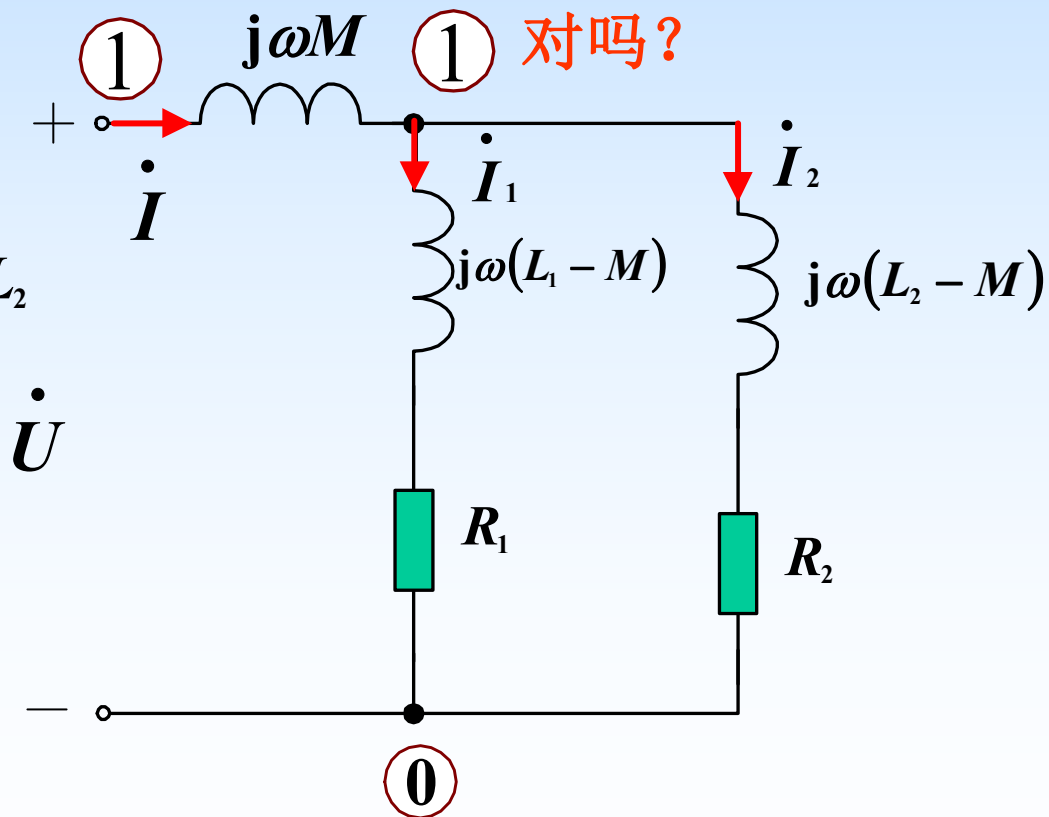
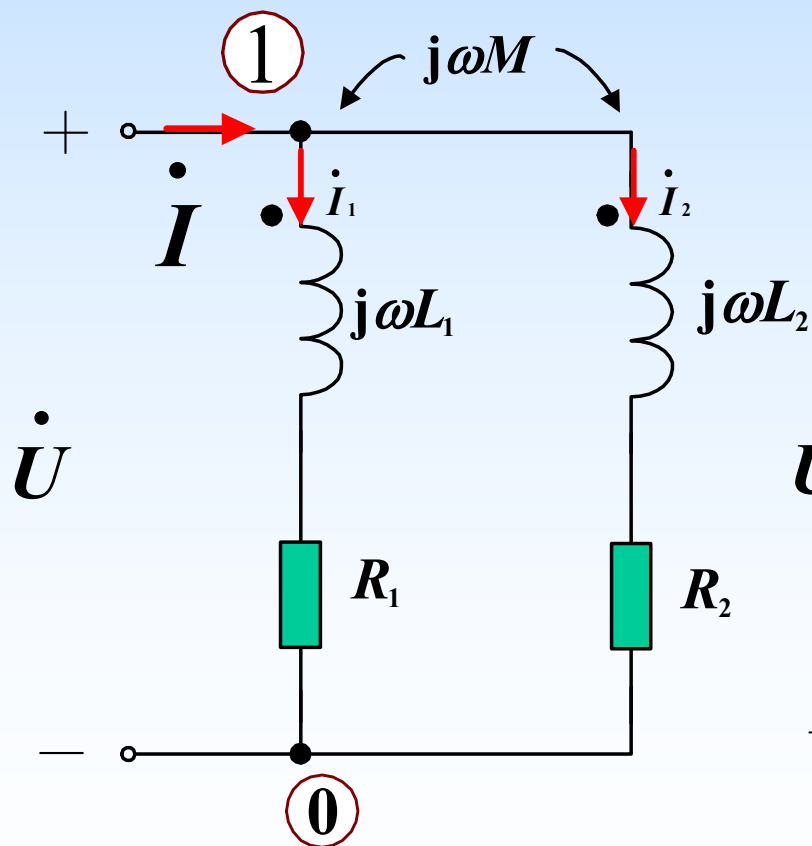
同名端在同一侧，称为同侧连接。

同名端不在同一侧，称为异侧连接。

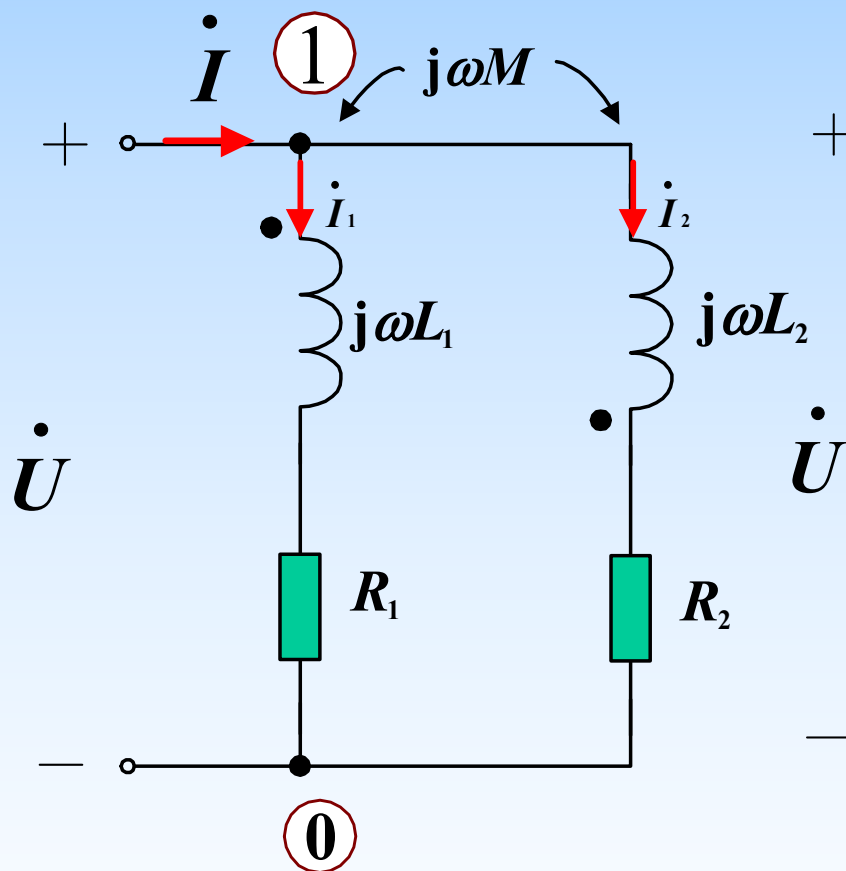
(a) 同侧连接

去耦以后注意结点位置。

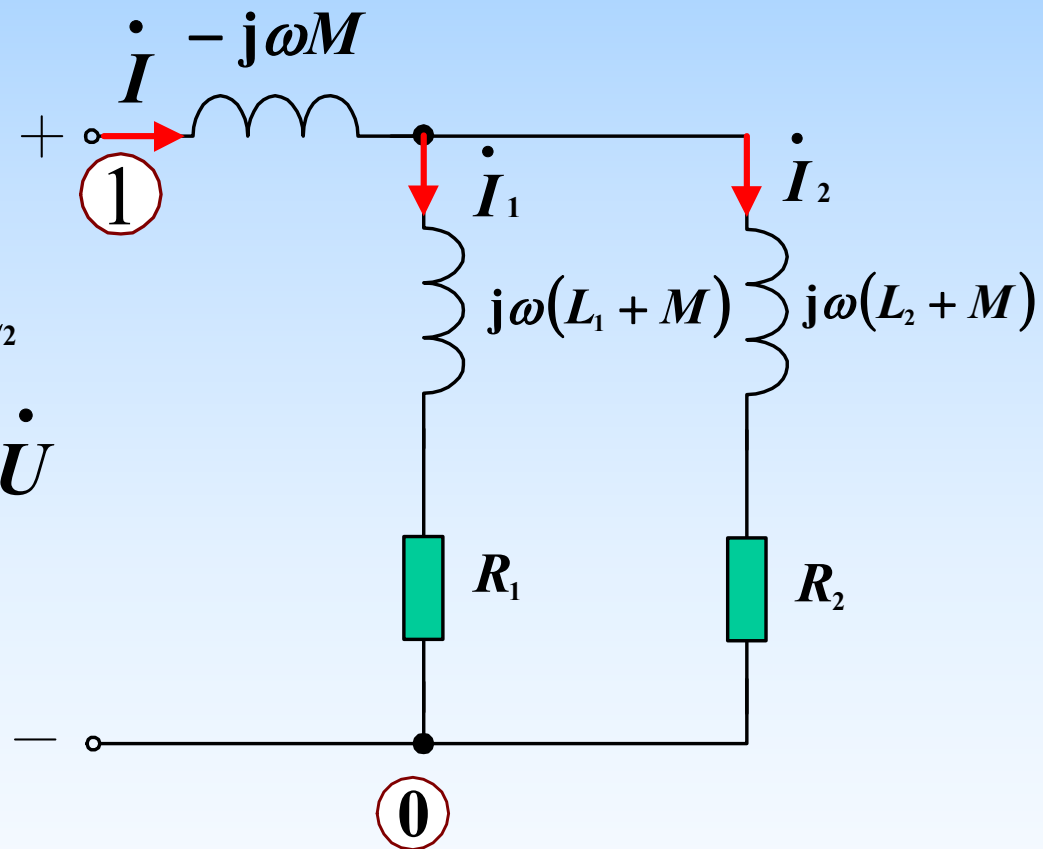
去耦等效电路



(b) 异侧连接

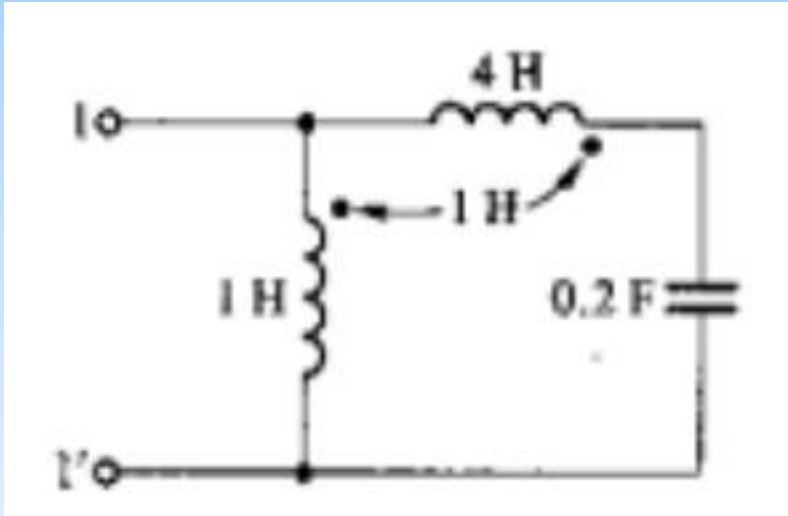


去耦等效电路

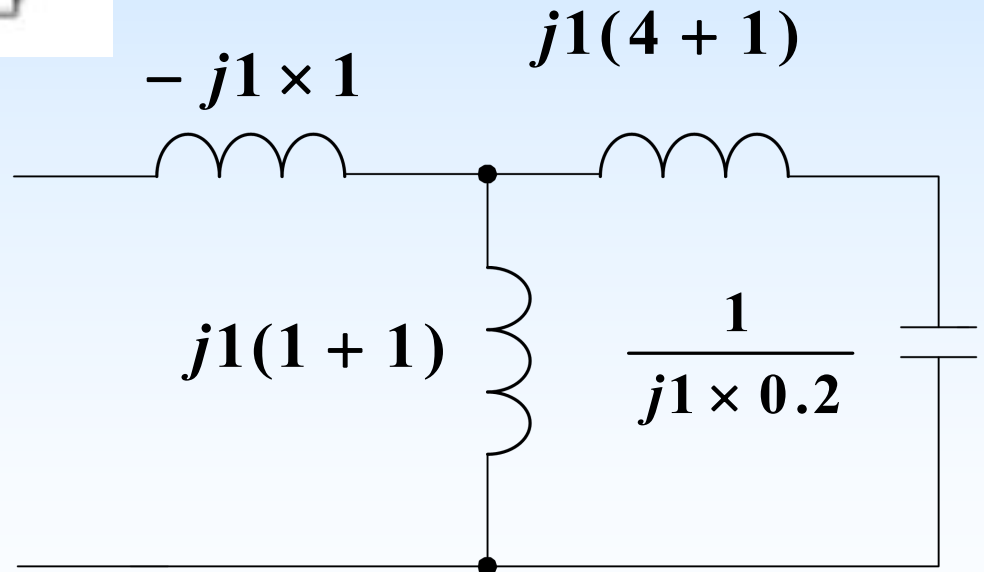


去耦以后注意结点位置。

求输入阻抗。 $\omega=1\text{rad/s}$



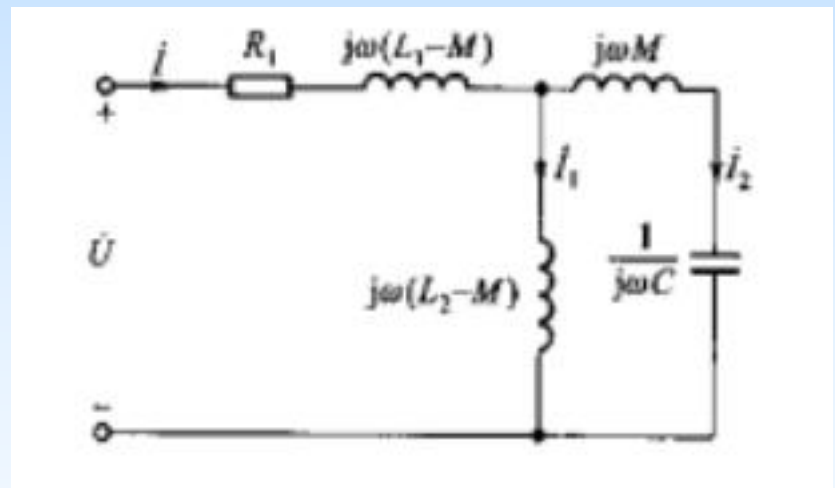
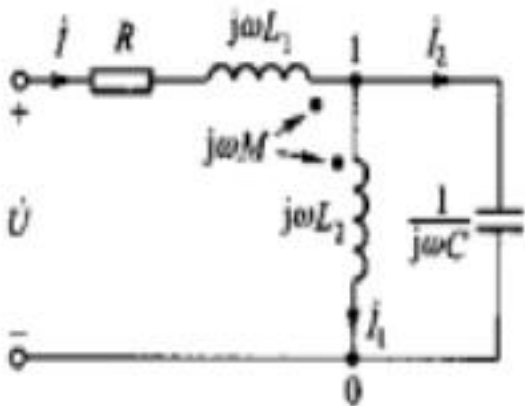
$$Z = -j1\Omega$$



10-14 电路如图，已知正弦电压源 $\dot{U} = 500\angle 0^\circ \text{ V}$

$\omega = 10^4 \text{ rad/s}$, $R = 50\Omega$, $L_1 = 70\text{mH}$, $L_2 = 25\text{mH}$, $M = 25\text{mH}$, $C = 1\mu\text{F}$,

求各支路电流。



由于 $L_2 - M = 0$ 最右边支路被短路

$$\dot{I} = \dot{I}_1 = \frac{\dot{U}}{R_1 + j\omega(L_1 - M)} = \frac{500\angle 0^\circ}{50 + j450} = 1.1\angle -83.7^\circ \text{ A}$$

$$\dot{I}_2 = 0$$